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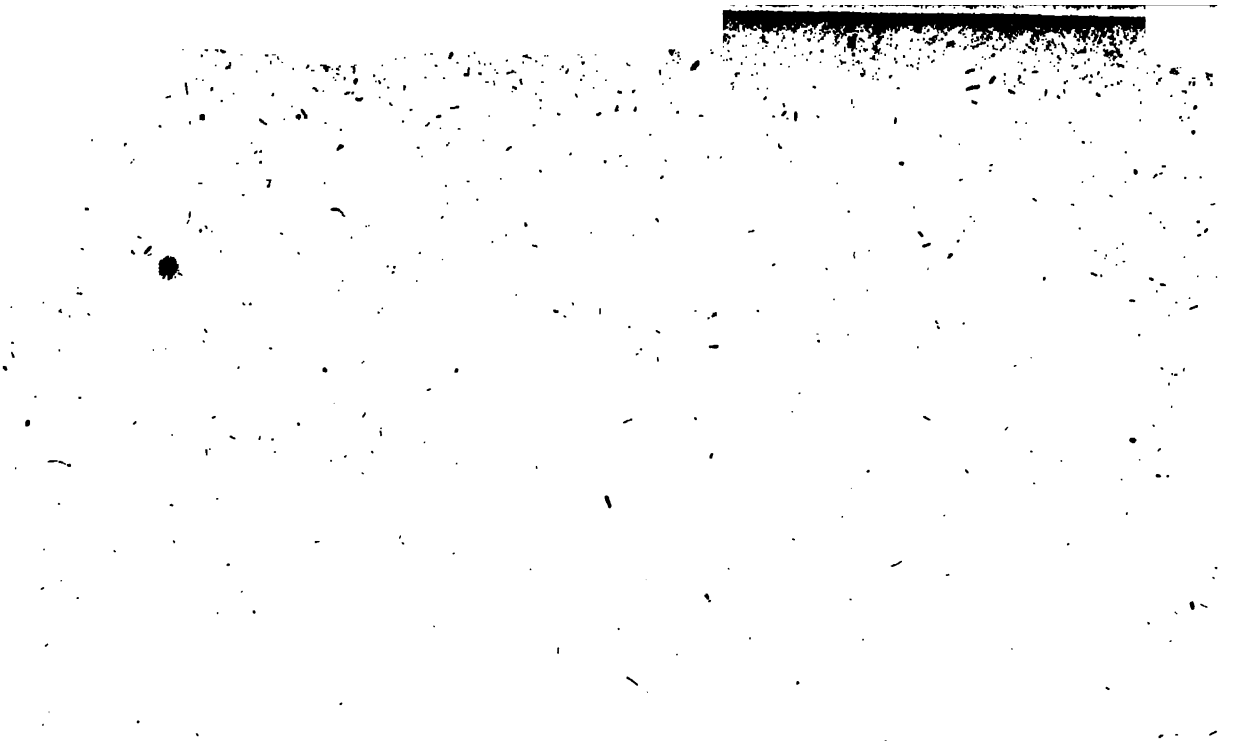
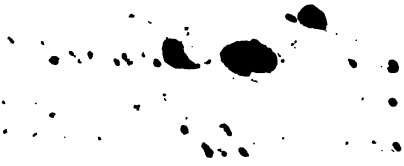


Mrs Sanford
New York October 28th



THE

PRACTICAL

ARITHMETIC:

IN WHICH THE

PRINCIPLES OF OPERATING BY NUMBERS

ARE

ANALYTICALLY EXPLAINED

AND

SYNTHETICALLY APPLIED:

TO WHICH IS APPENDED

A CONCISE ACCOUNT OF THE MENSURATION OF SUPERFICES AND SOLIDS, CHRONOLOGICAL PROBLEMS, THE MECHANICAL POWERS, &c. &c.

ADAPTED TO THE USE OF

SCHOOLS AND ACADEMIES

IN THE UNITED STATES.

BY T. H. BABCOCK.

"——Si quid novisti rectius istis,
Candidus imperti, si non, his utere mecum."—HORACE.

NEW-YORK.
G. & C. & H. CARVILL, 108 BROADWAY.
SEPT. 1829.

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Southern District of New-York, ss.

BE IT REMEMBERED, That on the 17th day of September, in the fifty-fourth year of the Independence of the United States of America, Anno Domini 1829, TERTULLUS H. BABCOCK of the said District, has deposited in this office the title of a book, the right whereof he claims as Proprietor, in the words following, to wit:

"The Practical Arithmetic; in which the principles of operating by numbers are analytically explained and synthetically applied: to which is appended, a concise account of the mensuration of superfices and solids, chronological problems, the mechanical powers, &c. &c.—adapted to the use of schools and academies in the United States.—by T. H. Babcock.

"—— Si quid novisti rectius istis,
Candidus imperti, si non, his utere mecum"—HORACE.

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FRED. J. BETTS.
Clerk of the Southern District of New-York

H. C. Sleight, Printer,
No. 3, Marble Building, Chatham Square, New-York.

PREFACE.

THE PLAN of the following pages was determined upon, and part of the work executed in the beginning of the year 1827. Since then the author has met with several new publications, claiming a great share of merit, and agreeing in many respects with his own views of the subject.

But, with reference to these, and of course to all others preceding, he presumes the following will be found to contain some very valuable improvements. In preparing it, however, it has been his steady and undeviating principle never to innovate, except with a view to improve.

He submits his work as containing more practical and necessary matter, the most important parts being exhibited more plainly to view, and the whole displayed in a manner more clear and interesting, than any heretofore published.

Feeling, however, a just diffidence, arising from a consciousness of the imperfection of all things sublunary, he solicits the indulgence of the public, and their FAVOUR and co-operation in furnishing him hints by which to meet, if possible, their more critical requisitions.

*** The THEORETICAL QUESTIONS, inserted in the work, are for the purpose of recitation. These, the learner should be required to answer APPROPRIATELY; that is, in a manner suited to the questions: and he should be apprised of this when he has his lessons given him. Thus, in answer to the question—"Why is it that figures in combination have a TENFOLD relation to one another?" instead of repeating it as in the book, he should reply, *because we have* TEN FIGURES *in our notation*; or in some other way in his own language. This, it is believed, will have the good effect of instructing him to form HIS OWN ANSWERS, and thus furnish an exercise in COMPOSITION. It is, therefore, to be hoped that no teacher will neglect these questions; but, on the contrary, that they will be thoroughly and punctually attended to, whenever they occur. It may be well too, for the teacher to propose other questions; as, in the illustrations, it is very desirable that every item should be pointed out, and the learner questioned, until he is thought to understand it fully and clearly.

The use of the Almanac on the following page, is to find the *day of the week* answering to any given day of ANY MONTH.—RULE.—Look for the day against the proposed year, then at the top, under the given month, you will find *the* DAYS OF THE MONTH *on which* THAT DAY *falls*.

EXAM. Com. Perry was born Aug. 21, 1785, on what day of the week was that?

Against the proposed year we find Tuesday, and at the top under the given month, we find, that Tuesday is the 2d, 9th, 16th, and 23d; therefore, the 21st falls on *Sunday*, the day required.

By this Almanac may also be found the DOMINICAL LETTER and EPACT of any year for ONE CENTURY, from the commencement of the AMERICAN INDEPENDENCE.

PERPETUAL ALMANAC.

Feb. March Nov.	Feb.* Aug.	May	Jan. Oct.	Jan.* April July	Sept. Dec.	June
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

N. B. In leap year, Jan. and Feb. must be taken in the columns marked *

Y's.	Days	Dom. Let.	Ept.	Y's.	Days	Dom. Let.	Ept.	Y's.	D's.	Dom. Let.	Ept.	Y's.	D's.	Dom. Let.	Ept.
'76	Frid.†	GF	9	'01	Sun.	D	15	'26	W.	A	22	'51	St.	E	28
'77	Sat.	E	20	'02	Mon.	C	26	'27	Th.	G	3	'52	M.†	DC	9
'78	Sun.	D	1	'03	Tues.	B	7	'28	St.†	FE	14	'53	Tu.	B	20
'79	Mon.	C	12	'04	Thur.†	AG	18	'29	S.	D	25	'54	W.	A	1
'80	Wed.†	BA	23	'05	Frid.	F	29	'30	M.	C	6	'55	Th.	G	12
'81	Thur.	G	4	'06	Sat.	E	11	'31	Tu.	B	17	'56	St.†	FE	23
'82	Frid.	F	15	'07	Sun.	D	22	'32	Th.†	AG	28	'57	S.	D	4
'83	Sat.	E	26	'08	Tues.†	CB	3	'33	F.	F	9	'58	M.	C	15
'84	Mon.†	DC	7	'09	Wed.	A	14	'34	St.	E	20	'59	Tu.	B	26
'85	Tues.	B	18	'00	Thur.	G	25	'35	S.	D	1	'60	Th.†	AG	7
'86	Wed.	A	29	'11	Frid.	F	6	'36	Tu.†	CB	12	'61	F.	F	18
'87	Thur.	G	11	'12	Sun.†	ED	17	'37	W.	A	23	'62	St.	E	29
'88	Sat.†	FE	22	'13	Mon.	C	28	'38	Th.	G	4	'63	S.	D	11
'89	Sun.	D	3	'14	Tues.	B	9	'39	F.	F	15	'64	Tu.†	CB	22
'90	Mon.	C	14	'15	Wed.	A	20	'40	S.†	ED	26	'65	W.	A	3
'91	Tues.	B	25	'16	Frid.†	GF	1	'41	M.	C	7	'66	Th.	G	14
'92	Thur.†	AG	6	'17	Sat.	E	12	'42	Tu.	B	18	'67	F.	F	25
'93	Frid.	F	17	'18	Sun.	D	23	43	W.	A	29	'68	S.†	ED	6
'94	Sat.	E	28	'19	Mon.	C	4	'44	F.†	GF	11	'69	M.	C	17
'95	Sun.	D	9	'20	Wed.†	BA	15	'45	St.	E	22	'70	Tu.	B	28
'96	Tues.†	CB	20	'21	Thur.	G	26	'46	S.	D	3	'71	W.	A	9
'97	Wed.	A	1	'22	Frid.	F	7	'47	M.	C	14	'72	F.†	GF	20
'98	Thur.	G	12	'23	Sat.	E	18	'48	W.†	BA	25	'73	St.	E	1
'99	Frid.	F	23	'24	Mon.†	DC	29	'49	Th.	G	6	'74	S.	D	12
'8*	Sat.	E	4	'25	Tues.	B	11	'50	F.	F	17	'75	M.	C	23

*, *18 hundred.* †††, &c., distinguish the leap-years.

FORMS OF NOTES, &c.

NUMBER I.

$63.54

New-York, Sept. 17, 1822.

For value received, I promise to pay to OLIVER DEXTER, or order, sixty-three dollars fifty-four cents, on demand, with interest after three months.

WILLIAM TRUSTY.

Attest, TIMOTHY TESTIMONY.

NUMBER II.

$23.75

Albany, Sept. 19, 1827.

For value received, I promise to pay to A. B., or bearer, twenty-three dollars seventy-five cents, three months after date.

PETER PENCIL.

NUMBER III.

$17.94

Utica, July 4, 1828.

For value received, we, jointly and severally, promise to pay to C. D., or order, seventeen dollars ninety-four cents, on demand, with interest.

EZRA CARY.

Attest, TRUMAN ENOS.

JAMES DEAN.

OBSERVATIONS.

1. No note is negotiable unless the words "*or order*," otherwise "*or bearer*," be inserted in it.
2. If the note be written to pay him "*or order*," (No. I,) then the holder may endorse his note, that is, write his name on the back, and pass it to whom he pleases. Then A, who takes the note, calls on William Trusty for payment, and if he neglects, or is unable to pay, A may recover it of the endorser.
3. If a note be written to pay him "*or bearer*," (No. II,) then any person who holds the note, may sue and recover the same.
4. The rate of interest being established by law, it will be unnecessary to mention it in writing notes or obligations.
5. If a bond or note mention no time of payment, it is understood to be on demand, whether the words "*on demand*" be expressed or not.
6. All notes, payable at a certain time, are on interest as soon as they become due, though in such notes there be no mention of interest.
7. A note on demand, without any mention of interest, draws interest immediately after a demand is made.
8. If a note be given for a specific article, payable in any certain time, and the signer of such note suffers the time to elapse without delivering such article, the holder of the note will not be obliged to take the article afterwards, but may demand and recover the value of it in money.

FORM OF A BOND.

A bond, with a condition, from one to another.

Know all men by these presents, that I, C. D., of ——, in the county of ——, am held and firmly bound to E. F., of ——, in the sum of two hundred dollars, to be paid to the said E. F., or his certain attorney, his executors, administrators, or assigns, to which payment, well and truly to be made, I bind myself, my heirs, executors, and administrators, firmly by these presents. Sealed with my seal. Dated the eleventh day of ——, *in the year of* our Lord one thousand eight hundred and two.

THE CONDITION of this obligation is such, that, if the above-bound C. D., his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the above-named E. F., his executors, administrators, or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of —— next ensuing the date hereof,—then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

NOTE. The principal difference between a note and a bond is, that the latter is an instrument of more solemnity, being given under seal. Also, a note may be controlled by a special agreement, different from the note; whereas, in case of a bond, no special agreement can in the least control what appears to have been the intention of the parties, as expressed by the words in the condition of the bond.

RECEIPTS.

Washington, Sept. 19, 1827.

Received of Mr. OLIVER CLARK, ten dollars in full of all demands.

PETER PARLEY.

Philadelphia, Oct. 9, 1827.

Received of Mr. JAMES PAYWELL, five dollars in full of all accounts.

FRANCIS GRANGER.

Baltimore, Dec. 17, 1828.

Received of Mr. SIMON SIMPSON, by the hands of ERASTUS CRAFTS, sixteen dollars twenty-five cents in full of all demands, witness my hand.

WILLIAM WALLACE.

ORDERS.

Pittsburgh, Jan. 4, 1829.

Mr. THOMAS CAREFUL, For value received, pay to A. B. or order, ten dollars, and place the same to my account. LEVI MARTIN.

Cincinnati, Aug. 4, 1829.

Mr. CYRUS BURGE, Please to deliver Mr. L. D. such goods as he may call for, not exceeding the sum of twenty dollars, and place the same to the account of your humble servant, NICHOLAS REUBENS.

BOOK KEEPING.

It is necessary that every man should have some regular, uniform method of keeping his accounts. What this method shall be, the law does not prescribe; but, in cases of dispute, it requires that the book, or that on which the charges were *originally* made, be produced in open court, when he will be required to answer to the following questions:

Is this your book, and the method in which you keep your accounts?

Did you make the charges now in dispute, at the time they purport to have been made? Are they just and true?

Have you received pay for them, or any part? if so, how much?

An answer in the affirmative, *under oath*, to the above questions, (the last only excepted,) is all that is required to substantiate his claim.

For farmers and mechanics, it will be found both convenient and easy, to have one single book; entering the name of the person with whom an account is to be opened, at the top of the left hand page *Dr.*, and at the top of the right hand page *Cr.* ☞ Any person on your book, is Dr. to what he receives, and *by what you receive.*

INDEX.

ARITHMETIC 9
Quantity 9
Number 9
Unit, unity, or one 9
Notation 9
Numeration 12
Addition of simple numbers 14
Subtraction of simple numbers 17
Multiplication of do. 20
Division 26
——— Fractions arise from division 32
Miscellaneous questions in the foregoing rules . . . 34
FRACTIONS—Vulgar 36
Proper, Improper, &c. 36
Reduction of fractions 37
To reduce a Mixed Number to an Improper Fraction . . 37
——— a Whole Number to the form of a Fraction . . 37
Value of a fraction 38
To reduce an Improper Fraction to its value 38
Effect of multiplying the Numerator 38
Effect of multiplying the Denominator 38
Effect of multiplying both Numerator and Denominator . . 39
To multiply by a fraction 39
To multiply a Fraction by a Whole Number 40
To multiply by a Mixed Number 40
To multiply a Mixed Number by a Whole Number . . . 41
To divide by a Fraction 42
To divide a Fraction by a Whole Number 42
To divide by a Mixed Number 42
To divide a Mixed Number by a Whole Number . . . 43
Compound Numbers 44
Reduction 45
Addition of compound numbers 55
Subtraction 57
Multiplication and Division 59
Fractions, continued from page 44 63
Reduction to their lowest terms 63
Greatest Common Measure, how found 64
To multiply fractions by fractions 64
To divide one fraction by another 66
Addition and subtraction of fractions 66
To reduce fractions to a common denominator . . . 67
Least common multiple, how found 68
To find the new numerators 68
To add or subtract fractions 69
Decimal fractions 70
Federal money 72
Addition and subtraction of decimals 73
Multiplication of decimals 74
Division of do. 75
Reduction of vulgar fractions to decimals 77
Reduction of compound numbers to decimals 78
Reduction of currencies 80
Ratio, or the relation of numbers 83

Proportion, or Single Rule of Three 83
Compound Proportion, or Double Rule of Three 86
Practice 90
Interest 91
Time, Rate per cent., and Amount given to find the Principal . 95
To find the Prime Cost of goods, having their gain or loss per cent. &c. 95
Discount 95
Time, Rate per cent., and Interest given, to find the Principal . 96
Principal, Interest, and Time being given, to find the rate per cent. 96
The prices at which goods are bought and sold being given to find the gain or loss *per cent.* 97
Principal, Rate per cent., and Interest given, to find the Time . 97
Commission 98
Insurance 98
Buying and selling stock 98
Loss and Gain 99
To compute interest on notes, bonds, &c. when partial payments have been made 99
Compound interest 100
Equation of Payments 103
Fellowship 104
Taxes, method of assessing 105
Double Fellowship 106
Alligation 107
Position 111
Permutation and Combination 114
Duodecimals 115
Involution, or the Raising of Powers 117
Evolution, or the Extraction of Roots 118
Extraction of the Square Root, ... 119
Application and use 123
Extraction of the Cube Root 124
Application and use 127
Arithmetical Progression 128
Geometrical Progression 131
Compound Interest by Progression 133
Annuites at Compound Interest 135

MISCELLANEOUS MATTER.

Chronological Problems 139
Gravities of Bodies 140
Specific gravities 141
Fall of Bodies 144
Mechanical Powers 147
Pendulums 148
Mensuration of Superfices 149
Mensuration of Solids 155
Guaging 158
Algebra 159
Trigonometrical and other surveying 159
Miscellaneous Questions 173

NOTE. It is presumed that no one will be so prejudiced as to object to the singular verb, in—"3 and 4 is 7," "7 and 5 is 12," "3 is to 4 as 12 to 16," &c. These are elliptical expressions; the word *number*, *result*, or *ratio*, being always understood as the nominative.

ARITHMETIC.

ART. 1. ARITHMETIC is the SCIENCE OF NUMBERS, and the ART OF USING THEM.

It is a SCIENCE when it explains the various properties and relations of numbers, and an ART when it merely exhibits their practical operations.

2. QUANTITY is a RELATIVE IDEA, that may be referred to any thing that is susceptible *of being* MORE *or* LESS.

As, for instance, to—*orange, lemon, sugar, coffee, tea*, &c.

Quantity is said to be DISCRETE when applied to distinct objects, as consisting of ONE OR MORE THAN ONE. This kind of quantity is also called *number*. Such are *one orange, two oranges, three lemons*, &c.

3. NUMBER is that which is used to express the RELATIONS of *quantity*.

As ONE *orange*, TWO *lemons*, THREE *hogsheads*, FOUR *bags*, &c.

When number is used without reference to any particular object, it is called ABSTRACT. Thus, when we say ONE, TWO, THREE, &c., without referring these expressions to any particular object, we use them in an *abstract manner*.

4. A UNIT, UNITY, or ONE, is the beginning of number, and signifies one single thing of any kind.

As *one* man, *one* barrel, *one* hogshead, &c.

The terms, *unit, unity*, or *one*, may, however, be applied to *any number of ones* when used COLLECTIVELY; as *a dozen, a score, a hundred, a thousand*, &c.

NOTATION.

5. NOTATION is a method of denoting number by characters.

Various methods have been devised for this purpose, but only TWO are at present used. These are the *Roman*,* and the *Arabic*, or *Indian method*.

---

* The Roman method makes use of these seven letters, I, V, X, L, C, D, M; I represents *one*, V *five*, X *ten*, L *fifty*, C *one hundred*, D *five hundred*, and M *one thousand*.

As often as any letter is repeated, so often its value is repeated; unless it be a letter representing a *less* number placed before one representing a *greater*; in which case the less number is to be taken from the greater. Thus, IV represents *four*, IX *nine*, XL *forty*, &c.

One I.	Thirty XXX.	Five Hundred D or IↃ.*
Two II.	Forty XXXX or XL.	Six Hundred DC.
Three III.	Fifty L.	Seven Hundred DCC.
Four IIII or IV.	Sixty LX.	Eight Hundred DCCC.
Five V.	Seventy LXX.	Nine Hundred DCCCC.
Six VI.	Eighty LXXX.	One Thousand M or CIↃ.†
Seven VII.	Ninety LXXXX or XC.	Five Thousand IↃↃ or $\overline{V}$.‡
Eight VIII.	One Hundred C.	Ten Thousand CCIↃↃ or $\overline{X}$.
Nine VIIII or IX.	Two Hundred CC.	Fifty Thousand IↃↃↃ.
Ten X.	Three Hundred CCC.	Hund. Thousand CCCIↃↃↃ or $\overline{C}$.
Twenty XX.	Four Hundred CCCC.	One Million $\overline{M}$.

* Used instead of D. For every additional Ↄ, the number is increased *ten times*.

† Every C and Ↄ, put at each end of CIↃ, increases its value ten times.

‡ A line over any number increases its value one thousand times.

6. The ARABIC, or INDIAN METHOD is thatwhich is employed in CALCULATIONS,* and is comprised in the use of these TEN characters:

One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.	Nought, or Cypher.†
1	2	3	4	5	6	7	8	9	0

7. These characters are, in general, called FIGURES or DIGITS.‡

8. The first nine are called SIGNIFICANT FIGURES, because they always *signify something;* and the last NOUGHT or CIPHER, because it signifies *nothing*.

9. The first figure (1) denotes one single thing or object, and the other *significant figures* denote in regular succession each one thing or object more than the preceding.

10. If we consider these figures attentively, we shall perceive that no number higher than 9 can be expressed by *any* ONE *of them*. They are, therefore, distinguished into ORDERS, and made to signify differently, as they are placed in the 1st, 2d, 3d, &c. order, beginning at the RIGHT HAND, and proceeding towards the LEFT.

The next number higher than 9, is called TEN; to denote which we use the same character (1) that we do to represent what is called a *simple unit;* but to distinguish it from that, we place it on the left hand of a 0; thus 10. The 0 fills the place of units of the FIRST ORDER or of *simple units*, and the 1 that of units of the SECOND ORDER OF ORDER OF TENS.

[It is to be regreted that the names *eleven* and *twelve*, (as seen on the following page,) have obtained, for it would certainly be more systematic to call one and ten *oneteen*, two and ten *twoteen*, three and ten *threeteen* or *thirteen*, &c. It will be seen that there are *three names* for TEN, viz., *ten*, *teen*, and *ty;* TEEN as in thir*teen*, four*teen*, &c.; TY as in twen*ty*, thir*ty*, &c.]

---

* Derived from the Latin *calculare*, to calculate. *Calculare* is derived from *calculus*, which signifies a *small stone;* as it is supposed the ancients used to perform their operations by means of small stones or pebbles.

† These figures were formerly all called *ciphers*, hence it came about that the art of arithmetic was called CIPHERING.

‡ Derived from the Latin *digitus*, which signifies a finger; as the ancients used to count by their fingers instead of our counting by these characters.

1 1 1,1 1 0,1 0 0 &c.	One unit of the THIRD ORDER, One unit of the SECOND ORDER, One unit of the FIRST ORDER, } called ONE HUNDRED AND ELEVEN.
1 0 0,0 0 0 &c.	One unit of the THIRD ORDER, One unit of the SECOND ORDER, No units of the FIRST ORDER, } called ONE HUNDRED AND TEN.
1 0 0 &c.	One unit of the THIRD ORDER, called ONE HUNDRED.
3 0,2 0 0 &c.	No units of the SECOND ORDER, being the PLACE *of tens*.
3 0,2 0 1 &c.	No units of the SECOND ORDER, being the *place of simple units*. Here the figure 1 is removed two places to the left, that it may represent TEN TENS, or *one hundred*.
&c.	Three units of the SECOND ORDER, or ORDER OF TENS, called THIRTY.
&c.	No units of the FIRST ORDER, being the PLACE of *simple units*.
2 0 &c.	Two units of the SECOND ORDER, or ORDER OF TENS, called TWENTY.
2 0 1 &c.	No units of the FIRST ORDER, being the PLACE of simple units filled with a 0, to make the other figure 2 represent TENS, by removing it one place further to the left.
&c.	One unit of the SECOND ORDER, or ORDER OF TENS; equal to TEN *s. u.* } called THIRTEEN.
1 3,1 2 1 &c.	Three units of the FIRST ORDER, or *three simple units*
1 2 &c.	One unit of the SECOND ORDER, or ORDER OF TENS equal to TEN *s. u.* } called TWELVE.
1 2 1 &c.	Two units of the FIRST ORDER, or *two simple units*
1 1 &c.	One unit of the SECOND ORDER, or ORDER OF TENS equal to TEN *s. u.* } called ELEVEN.
1 1 0 &c.	One unit of the FIRST ORDER, or *one simple unit*
9 0,3 2 1 &c.	No unit of the SECOND ORDER, or ORDER OF TENS, equal to TEN *s. u.* filled with a 0, to make the other 1 represent TEN, by removing it one place further to the left.
&c.	Nine units of the FIRST ORDER, or *nine simple units*.
&c.	Three units of the FIRST ORDER, or *three simple units*.
&c.	Two units of the FIRST ORDER, or *two simple units*.
&c.	One unit of the FIRST ORDER, or *one simple unit*

11. These names may, however, be rendered more concise:

Units of the *first order*, for instance, may be called simply UNITS: units of the *second order*, TENS; units of the *third order*, HUNDREDS, &c.

Example I. Write THREE *hundred and* SIXTY-FIVE in figures.

In this number are three orders, viz., *units*, *tens*, and *hundreds*.

UNITS always occupy the *first place* at the right hand, (Art. 10,) TENS the *second* towards the left, HUNDREDS the *third*, and so on.

As, therefore, the number to be written, *separated into its parts*, contains *five* (units) represented by 5, *six* (tens) represented by 6 *in the second place*, and *three* (hundred) represented by 3 *in the third place*, it will be written thus:

Hund.	Tens.	Units.
3	6	5

Example 2. Write THREE *hundred and* FIVE in figures.

Here are TWO orders *expressed*, and ONE *understood*, which, if filled out, the number will stand: THREE *hundred* NO *tens and* FIVE. As, therefore, there are no *tens* in this case, we put 0 in the *place of tens*, and the number will be written thus:

Hund.	Tens.	Units.
3	0	5

Hence it appears that

12. FIGURES have *different values* according to the PLACE which they occupy.

Take for example the number, Hund. Tens. Units. 3 3 3 } made by the same figure three times repeated.

The 3 in the first place at the right hand, represents 3 units; the same figure in the *second place*, represents 3 *tens* or thirty; its value being there increased *ten times*. Again, the same figure in the *third place*, represents 3 hundreds, which is ten times the value of it in the place immediately preceding, that is, in the place of *tens*. This is common to every combination of figures.

Hence *we have this* FUNDAMENTAL LAW *in notation*, viz., that

13. EACH REMOVAL of place towards the LEFT HAND increases the *value of a figure* TEN TIMES.*

From the foregoing, it plainly appears, that the different values of a figure may be divided into

14. TWO KINDS, which may be called *Simple* and *Local*.†

I. The SIMPLE VALUE of a figure is what it represents when it stands alone or in unit's place.

II. The LOCAL VALUE is determined from the PLACE which it occupies.

Hund. Tens Units — Local Value — Local Value — Simple Value

EXAMPLE. 5 5 5, that is, 5 hundreds 5 tens and 5 (units), or *five hundred and fifty five*.

To write any proposed number in figures,

Agreeably to the principles now given, we have this

15. RULE. Write down the FIGURES as their values are expressed, and supply every deficiency in the order with CIPHERS.

EXAMPLE. Write FIVE *hundred and* FOUR in figures.

Answer. 5 0 (*tens*) 4, or 504.

Write *five thousand and four* in figures.

..... *five thousand and forty* .....

..... *four thousand and five* .....

..... *four thousand and fifty* .....

..... *six hundred and one* .....

..... *six thousand and seventy*. .....

NUMERATION

16. Is the method of *numerating* and *reading* numbers.

	Thou.	Hund.	Tens	Units	
For instance,					Is numerated
the number,	2	3	5	6,	and read 2 thousand 3 hund. and fifty-six

NOTE. For the more easy reading of large numbers, they are usually separated by a comma into half periods of *three figures each*, as in the following

* The reason of this tenfold relation originates in our using TEN figures in our notation. Any *other number* of figures might be used, but their relation to one another in combination must always be that indicated by the *number of figures*.

† From the Latin *locus*, which signifies *place*.

NUMERATION TABLE.

Billions,† Hundreds of thousands of millions,	Tens of thousands of millions,	Thousands of millions,	Hundreds of millions,	Tens of millions,	Millions,	Hundreds of thousands,	Tens of thousands,	Thousands,	Hundreds,	Tens,	Units,	Numerated.*	And read.
										2	1	Two tens and one, i. e., twenty-one.	
									3	2	1	Three hundred and twenty-one.	
								4,	3	2	1	Four thousand 321.	
							5	4,	3	2	1	Fifty-four thousand 321.	
						6	5	4,	3	2	1	Six hundred and fifty-four thousand	321.
					7,	6	5	4,	3	2	1	Seven millions 654 thousand 321.	
				8	7,	6	5	4,	3	2	1	Eighty-seven millions 654 thousand	321.
			9	8	7,	6	5	4,	3	2	1	Nine hundred and 87 millions 654 th	ousand 321.
		1	0,	9	8	7,	6	5	4,	3	2	1	Ten thousand 987 millions 654 thou	sand 321.
	2	1	0,	9	8	7,	6	5	4,	3	2	1	Two hundred and ten thousand 987	millions 654,321.
3,	2	1	0,	9	8	7,	6	5	4,	3	2	1	Three millions 210 thousand 987 mi	llions 654,321.

Hence, *To numerate any given number,*

17. Rule. Numerate from the right hand towards the left, saying *units, tens, hundreds,* &c. as in the table.

And, *To* read *the* same,

18. Rule. To the simple value of each figure, beginning at the left hand, join the *name of its place.*

Numerate and read the population of the city of

New-York, . . (1820) . .	123,706‡	Boston, ...	43,298
Philadelphia, ...	108,116	Baltimore, ...	62,738
Washington, ...	13,247	New-Orleans, ...	27,176

State of New-York, ... 1,372,812
State of Ohio, ... 581,434
United States of America, ... 9,636,423

THEORETICAL QUESTIONS.

What is Arithmetic? When is it called a *science?* When an *art?* What is quantity? When is it said to be *discrete?* What is number? When is number called *abstract?* What is a *unit, unity* or *one?* When may these terms be applied to *any number of ones?* What is notation? How many methods of notation are there? What are they? Which is employed in *calculations?* In what is it comprised? What are these characters, in general, called? What are the first nine called? Why? What the last? Why? What does the first figure denote? What the other significant figures? What appears from the combination of figures? What is the fundamental law in notation? How many kinds may the different values of a figure be divided into? What is the simple value of a figure? What the local value? What is the rule by which to write any proposed number? What is numeration? How do you numerate any given number? How do you read the same?

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* The circumstance of the increase of figures, and consequently of this numerating proceeding from the right hand towards the left, originates in the fact, that this method is borrowed from the Arabic or rather Asiatic nations, who have the habit of writing from the right towards the left.

† After Billions proceed Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, &c.

‡ New-York now probably contains near 200 thousand.

Numerate and read the following, being all the PERFECT NUMBERS* *known*

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639229258349412352
33550336	9903520314282971830448816128

ADDITION OF SIMPLE NUMBERS,

COMMONLY CALLED

SIMPLE ADDITION.

19. SIMPLE NUMBERS are numbers of the SAME NAME.

As 13 dollars, 27 pounds, 36 guineas, &c.

20. SIMPLE ADDITION is finding the SUM of *two* or *more* SIMPLE NUMBERS.

Illus. 1. Little Edwin has three apples in one pocket, 5 in another, and 7 in his hat; how many apples has he in all?

Apples.

Tens.	Units.
.	3
.	5
.	7
1	5

Answer. 15 Apples.

Here it is plain that we find the answer, for 7 and 5 is 12, and 3 is 15. This separated into its parts, becomes 1 ten and 5 units. The unit figure (5) is placed directly under the row of figures added, and the other figure (1) is removed one place further to the left, that it may represent TEN.

2. How much money in 3 bags, the first containing 253 dollars, the second 345, and the third 937?

Dollars.

Thou.	Hund.	Tens	Units
.	2	5	3
.	3	4	5
.	9	3	7
.	.	1	5
.	1	2	.
1	4	.	.
1	5	3	5

Answer. 1535 Dol.

First, we set the numbers down with *units* under *units*, *tens* under *tens*, &c.: Then we begin at the right hand, and add up each row, setting the results underneath as before directed. It will be seen from the operation, that whenever the sum of any row exceeds what can be expressed by a single figure, (Art. 10,) we have to place the figure coming to the left, under the next higher order; and these figures are added in, when adding up the row of that higher order. Hence these two additions may be performed at once: For, we may set down the unit figure of the result of each row, and keep in mind the number of TENS to add in adding up the next left hand row. This adding for the *number of tens* is called *carrying for every ten*.† Proceeding in this way, we observe the process as expressed in the following language:

253
345
937
1535

After setting the numbers down as before directed, we begin at the right hand and add up the row of units, saying 7 and 5 is 12, and 3 is 15; set down 5 and carry 1 . . . . . to 3 is 4, and 4 is 8, and 5 is 13; set down 3 and carry 1 . . . . . to 9 is 10, and 3 is 13, and 2 is 15. Set down the whole sum.

* A perfect number is one that is equal to the sum of all its ALIQUOT PARTS.

† We *carry one for every ten*, because "each removal of place towards the left hand increases the value of a figure ten times." The reason of this, and indeed of the whole algorithm of arithmetic, originates in the *number of figures* used in our notation.

21. Hence, for the *addition of simple numbers*, we have this

RULE I. Place UNITS under UNITS, TENS under TENS, &c.

II. Add up the row of units, set down the unit figure of the sum, and *carry* ONE *for every* TEN to the next left hand row or row of tens.

III. Do the same with each row, observing to set down the whole sum of the last.

William has 54 cents, Charles 76, and Henry 59: how many cents have all three? *Ans.* 189 cents.

How many acres in 3 farms, the first containing 575, the second 599, and the third 514 acres? *Ans.* 1688.

22. PROOF. Begin at the TOP and add *downward*.

Let the scholar now answer to the following questions.

What is the sum of 2 and 4? 4 and 6? 6 and 8? 8 and 9? 3 and 5? 5 and 7? 7 and 8? 4 and 5? 5 and 6? 6 and 7? 7 and 8? 5 and 8? 6 and 9? 5 and 9? 3 and 8? 2 and 9? &c.*

23. This sign = signifies EQUALITY: and is read *equals*, or *is equal to*.

As 3 and 5 = 8, that is to say, 3 and 5 *equals* 8, or *is equal to* 8.

24. This sign + signifies ADDITION, and is read *and*, *plus*,† or *more*.

As 3 + 5 = 8, that is to say, 3 and 5, or 3 *plus* 5, or 3 *more* 5, is equal to 8.

EXAMPLES IN ADDITION.

1. A gentleman bought a chaise for 217 dollars, a horse for 96, and saddle for 15; how much did the whole come to? *Ans.* 328 dollars.

2. How much in 3 bags of money, one containing 756 dollars, another 817, and a third 989? *Ans.* 2562 dollars.

3. Bought 5 loads of hay, weighing as follows, viz. 1st load 2475 pounds, 2d 1964 pounds, 3d 1843 pounds, 4th 2276 pounds, 5th 1728 pounds; how much in all? *Ans.* 10286 pounds.

4. Money of gold and silver was first made at Argos 894 years before Christ; how long has it been in use?

5. Bought goods to the amount of 325 dollars; paid for truckage 2 dollars, freight 7 dollars, and my own expenses came to 19 dollars; what do they stand me in? *Ans.* 353 dollars.

6. Borrowed at different times, the following sums, viz., 267 dollars, 354 dollars, 75 dollars, 587 dollars; what is the amount? *Ans.* 1283 dollars.

7. St. Paul's Cathedral, in London, cost 800,000 pounds; the Royal Exchange 80,000; the Mansion House 40,000; Black Friars' Bridge 152,840; Westminister Bridge 389,000; and the Monument 13,000. Required the amount of these sums? *Ans.* 1,474,840 pounds.

8. If one of the fore quarters of an ox weigh 176 pounds, the other 169: one of the hind quarters 185 pounds, the other 193; the hide 84 pounds, and tallow 78 pounds; what is the whole weight of the ox. *Ans.* 885 lb.

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* The teacher may extend these at pleasure.

† A Latin word signifying *more*.

9. A gentleman left to his daughter, 14 thousand 14 hundred and 14 dollars; he left to his son 1735 dollars more than to his daughter; what was the son's portion? and what the whole estate?

Ans. { Son's portion 17,149 dollars. Whole estate 32,563 . . . .

THEORETICAL QUESTIONS.

What are simple numbers? What is simple addition? What is the first part of the RULE? What the second?——the third? Why do you carry one for every ten? Why is it that figures in combination have a *tenfold* relation to one another? What is the method of PROOF? What is the sign of EQUALITY? How is it read? What the sign of ADDITION? How is the sign of addition read? What is *plus*?

EXAMPLES FOR PRACTICE.

1. SIR ISAAC NEWTON was born in the year 1642, he lived to be 85 years of age; in what year did he die? *Ans.* 1727.

2. George Washington was born 1732, he lived to the age of 67; in what year of our Lord did he die. *Ans.* 1799.

3. Benjamin Franklin was born 1706, he lived to the age of 84; in what year did he die? *Ans.* 1790.

4. Homer was born 733 years before Christ; how many years since?

5. Bought sugar to the amount of $327, Coffee $538, and tea $749; what is the amount? *Ans.* $1614.

6. In the city of Pekin in China, is a bell weighing 120,000 pounds; at Nankin in the same empire, another of 50,000 lbs.; and at Erfort in Upper Saxony, the great bell weighs 25,400 lbs.; quere, the weight of the three. *Ans.* 195,400 lbs.

7. From the creation to the departure of the Israelites from Egypt was 2513 years; to the siege of Troy 307; to the building of Solomon's Temple 180; to the building of Rome 251; to the expulsion of the kings from Rome 244; to the destruction of Carthage 363; to the death of Julius Cæsar 102; to the Christian era 44; required the time from the Creation to the Christian era. *Ans.* 4004 years.

8. The population of the New England states in 1820 was as follows:—Maine, 298,335; New Hampshire, 244,161; Vermont, 235,764; Massachusetts, 253,287; Rhode Island, 83,059; Connecticut, 275,248; what was the whole number of inhabitants in those states, at that time. *Ans.* 1,389,854.

9. The distance from Portland, in the province of Maine, to Boston is 125 miles, from Boston to New-York 250, from New-York to Philadelphia 95, from Philadelphia to Baltimore 102, from Baltimore to Richmond in Virginia 176, from Richmond to Charleston, S. Carolina 540, and from Charleston, to Savannah, in Georgia 119; what is the distance from Portland to Savannah? *Ans.* 1507.

10. Thirty days are in September,
In April, June, and in November;
February hath twenty-eight alone,
And all the rest have thirty-one.
Now sum them up and let us hear
How many days are in the year. *Ans.* 365.

SUBTRACTION OF SIMPLE NUMBERS,

COMMONLY CALLED

SIMPLE SUBTRACTION.

25. SIMPLE SUBTRACTION is finding the DIFFERENCE between TWO SIMPLE NUMBERS.

Illus. 1. William had 78 peaches, and Henry 36; how many had William more than Henry?

William had 78	Here it is plain that we find the difference, for 6 (*units*) from 8 (*units*) evidently leaves 2 (*units*); and 3 tens from 7 (tens as evidently leaves 4 tens: therefore 4 tens and 2 (*units*) comprise the whole *difference* 42, the answer.
Henry " 36 Ans.	
Difference 42 peaches.	

2. After William had lost 3 of his peaches, how many had he then more than Henry?

	Tens.	Units.	
William had	7	5	left.
Henry "	3	6	Ans.
Difference	3	9	peaches.

Here we cannot take 6 (the unit figure of the lower number) from 5 (the unit figure of the upper); yet it is nevertheless evident, that we can take 36 from 75. To do this, however, we shall have to separate 75, into its parts, 7 tens and 5 (*units*): or, *borrowing* (as it is called) 1 TEN from the 7, as seen below, and adding it to 5 (the figure above), we separate it into 6 tens and 15 units. Then separating 36 into its parts, 3 tens and 6 units, we may begin and take 6 units from 15 units, by which we obtain a difference of 9 (*units*). Proceeding to the next left hand place, it is evident that we must either make the upper figure less by 1 (that we borrowed from it) or carry (as it is called) 1 to the next lower figure, and take the SUM from the upper. It is plain that either of these two modes will give the same result: For 3 from 6 (the upper figure made less by 1) leaves 3, the same as 4 from 7. But the operation (6 from 15) may be made more easy; for we may evidently take the lower figure from the 1 TEN that we borrowed and add in the upper figure, saying, in this case, 6 from 10 leaves 4, and 5 (the figure above) is 9; set down 9 and *carry* 1, &c.

	Tens.	and Units.	Tens.	and Units.
75=7	5	= 6	15	
36=		3	6	
Difference		3	9	

26. From the illustrations now given, we deduce the following general

RULE I. Place the *less number* under the *greater*, with UNITS under UNITS, TENS under TENS, &c.

II. Begin at the right hand, and take each figure in the lower line from the figure above:—

III. But, if the figure in the lower line EXCEED that above it, take it from ten, and add the remainder to the upper figure; set the result underneath, CARRY 1 to the next lower figure, and proceed as before.

27. The *upper number* is called the MINUEND, the *lower number* the SUBTRAHEND, and the *difference* the REMAINDER.

28. This sign — signifies SUBTRACTION, and is read *minus*,* or *less*.

As 7 — 2 = 5, that is to say 7 *minus* 2, or 7 *less* 2, is equal to 5.

What is 9 *minus* 5 *equal* to? 7—4? 5—3? 8—5? 8—3? 9—4? 9—3? 9—2? 7—5? 7—3? 7—2? 6—2? &c.

* A Latin word signifying *less*.

29. PROOF. Add the REMAINDER to the SUBTRAHEND, and if the work is right, THAT SUM will be equal to the MINUEND.

	Apples.	BASKET.
From	7,	the *minuend.*
Take	4,	the *subtrahend.*
	—	HAT.
Diff.	3,	the remainder,
	— IN THE BASKET.	
Proof	7= the minuend =	
the number in the HAT AGAIN.		

Edwin has 7 apples in a BASKET; he puts 4 of them in his HAT; how many remain in the BASKET?

If he put just 4 apples of the 7 in his HAT, it is very obvious, that he would leave 3 in the BASKET; and if he then put these 3 which remaine in the BASKET, also in his hat, he would obviously *diminish* the number of apples in the BASKET, to 0, and *increase* the number in his HAT to 7. But he would then have just as many apples in his HAT as he had before in the BASKET: Therefore, *adding the* REMAINDER *to the* SUBTRAHEND *will give a sum equal to the* MINUEND.

And it is therefore also obvious that

30. *Subtraction* is the REVERSE of *Addition.*

William had 9 cents, he gave James 4; how many had he then?
9 — 4 how many? *Ans.* 5.
James had 5 cents, William gave him 4; how many had he then?
5 + 4 how many? *Ans.* 9.
A man had 8 cows, he sold 3 of them; how many had he left?
8 — 3 how many? *Ans.* 5.
A man sold 3 cows, and had 5 left; how many had he at first?
3 + 5 how many? *Ans.* 8.

30 *a.* Hence also it appears that addition may be PROVED by subtraction, as well as subtraction by addition. That is, *by taking, successively, from the sum the* SEVERAL NUMBERS *which were added to make it;—If the work is right there will be* NO REMAINDER.

ADDITION.	SUBTRACTION, *Proof.*
7 + 5 + 3 = 15 sum.	15 — 7 — 5 — 3 = 0
9 + 7 + 5 + 3 = 24 sum, &c.	24 — 9 — 7 — 5 — 3 = 0, &c.
SUBTRACTION.	ADDITION, *Proof.*
8 — 6 = 2 Diff.	2 + 6 = 8, the minued.
7 — 3 = 4 Diff., &c.	4 + 3 = 7, ... &c.

30 *b.* And here it may be remarked, that EVERY OPERATION *in Arithmetic has its* OPPOSITE; and they are all founded upon the fundamental principle of INCREASE *and* DECREASE, *of* MORE *and* LESS, *or of* ADDITION *and* SUBTRACTION,

30 *c.* And it may be further remarked, that *when any* NUMBER *or* QUANTITY *is operated upon by* ANY RULE, and the result *by its* OPPOSITE *to the same extent, that number or quantity will remain* UNCHANGED.

10 + 5 = 15, and 15 — 5 = 10. 12 + 8 = 20, and 20 — 8 = 12.

A man had 8 cows, he bought 5 more, how many had he then?
8 + 5 how many? *Ans.* 13.
He afterwards sold 5, how many had he left?
13 — 5 how many? *Ans.* 8.

EXAMPLES IN SUBTRACTION.

1. Borrowed 817 dollars, paid 375; how much remains due?
Ans. 442 *dollars.*

2. A man bought 157 acres of land for 3617 dollars, of which he afterwards sold 79 acres for 2923 dollars; how much land has he left? and what does it stand him in?
Ans. He has 78 *acres left, and it stands him in* 694 *dollars.*

3. *What is the difference between* 717171, *and* 171717? *Ans.* 545454.

4. A man going into account with himself, finds his whole property amounts to 18406 dollars, and that he owes 10509 dollars; how does he stand? *Ans.* 7897 *dollars clear property.*

5. Bought goods to the amount of 7345 dollars, sold of the same, deducting profit, to the amount of 3579 dollars; what amount remains unsold? *Ans.* 3766 *dollars.*

6. The Arabic or Indian method of notation was brought into general use in Europe, about the year 970: how long since?

7. A merchant has to receive the following sums, viz., 819, 612, 705, 31, and 1756 dollars; he has to pay rent 150 dollars, and his note to the bank for 3789 dollars: Will he have money to borrow or to spare, and how much? *Ans. He will have to borrow* 16 *dollars.*

8. There are 365 days in a year, of which 52 are Sundays; how many working days in a year? *Ans.* 313

THEORETICAL QUESTIONS.

What is simple subtraction? What the first part of the RULE? the second? —— the third? What is the upper number called? —— the lower number? —— the difference? What is the SIGN of subtraction? How is it read? What is *minus?* What is the method of PROOF? What is obvious from the illustration of this method? What also appears from the examples that follow? How? What, after this, do you find TO BE REMARKED? What FURTHER?

EXAMPLES FOR PRACTICE.

1. Christopher Columbus landed in San Salvador 1492; King Charles was beheaded 1648, and his son restored in 1660; how many years between each of these events? *Ans.* 156, 12, *and* 168.

2. The mariner's compass was invented 1302, gunpowder 1344, and printing 1440; how many years between? *Ans.* 42, 96, *and* 138.

3. A man drew a TEN THOUSAND dollar prize, out of which he had to pay percentage 1500 dollars; how much had he left?

4. Two men set out from the same place at the same time; one goes 32 miles a day, and the other follows at the rate of 39 miles a day; how far will they be apart at the close of the first day? —— of the second? —— of the third? —— of the fourth? —— of the fifth?
Ans. to the last, 65 *miles.*

5. Three men bought a piece of land for 2356 dollars; the first man paid 1279 dollars, and the second 397 less than the first; how much did the second and third pay?
Ans. { *The second* 882 *dollars.* / *The third* 195 *dollars.*

6. There are two numbers, the greater is 9268, and their difference 597; what is the less? *Ans.* 8671.

7. A gentleman left, to his son and daughter, an estate of 32,563 dollars; he left to his son 17,149 dollars, and to his daughter the rest: What was the daughter's portion? *Ans.* $15,414

8. If a farm and the buildings on it be valued at $10,000 and the buildings alone at $4567, what is the value of the land? *Ans.* $5433.

9. How much must you add to 358,642 to make 1,487,945?
Ans. 1,029,303

10. Bought goods for $13,682, and sold them again for $15,293; did I gain or lose, and how much? *Ans. I gained,* $1611.

11. A man carried his produce to market; he sold his pork for $45, his cheese for $38, and his butter for $29; he received, in pay, salt to the value of $17, $10 worth of sugar, $5 worth of molasses, and the rest in money; how much money did he receive? Ans. $89.

12. In the city of Pekin in China, is a bell weighing, it is said, 120,000 *lbs.*; and at Nankin, in the same country, is another weighing 50,000*lbs.*; the first exceeds the great bell at Erfort, in Upper Saxony, by 94,600*lbs.*; how much then is the German bell inferior in weight to the second?

Ans. 24,600*lbs.*

MULTIPLICATION OF SIMPLE NUMBERS,

COMMONLY CALLED

SIMPLE MULTIPLICATION.

31. SIMPLE MULTIPLICATION is repeating any given *simple number* as often as there are UNITS in *another*; and is a short method of performing ADDITION.

Illus. 3 boys had 37 cents each; how many had they all?

BY ADDITION.

	Cents.
One boy had	37
Another	37
A third	37
All three	111 Cents.

Ans.

BY MULTIPLICATION.

Each boy had 37 cents.
All three 3 times
as many, or 111 cents.

Ans.

We multiply by 3, saying 3 times 7 is 21, set down 1 and carry 2; 3 times 3 is 9, and 2 we had to carry is 11. As multiplication is only a contraction of addition, it is plain, that we set down and carry the same as in that rule.

TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

32. This sign × signifies MULTIPLICATION, and is read *multiplied by*, or *into*.

As $3 \times 5 = 15$, that is to say, 3 *multiplied by* 5, or 3 *into* 5 is equal to 15.

33. There are THREE terms used in multiplication, namely, the *multiplicand*, *multiplier*, and *product*.

I. The MULTIPLICAND is the number given *to be multiplied*.

II. The MULTIPLIER is the number given to *multiply by*.

III. The PRODUCT is the number found from the operation.

Both *multiplicand* and *multiplier* are called FACTORS.

Illus. Oliver bought 3 books at 49 cents apiece; how much did they come to?

FACTORS.

cents.

Multiplicand $49 = 40 + 9$

Multiplier 3 no. of books.

$27 = 9 \times 3$

$12. = 40 \times 3$.

Product $147 = 49 \times 3$

Ans. Cents.

But this process may be abridged:

Thus—49 / 3 / Prod. 147

Here we multiply the 9 (units) by the 3, saying 3 times 9 is 27; set down 7, and carry 2. Then the 4 (tens,) saying 3 times 4 is 12, and 2 we had to carry is 14; set down 14.

NOTE. As the learner has committed the Table, it will be as easy to multiply by 12, as by any single figure, and the same process is to be observed as in this example. Hence,

34. *To multiply simple numbers,*

(When the multiplier does not exceed 12,)

RULE. Multiply EACH FIGURE in the *multiplicand* by the *multiplier*, observing *to set down and carry* as in ADDITION.

EXAMPLES.

1. James bought 5 yards of linen at 57 cents a yard; how much did it come to? *Ans.* 285 *cents.*
2. What will 7 cows come to at 18 dollars a head? *Ans.* 126 *dollars.*
3. What will 76 pounds of cheese come to at 8 cents a pound? *Ans.* 608 *cents.*
4. Robert sold 9 muskrat skins at 37 cents apiece, what did he get for them? *Ans.* 333 *cents.*
5. What will 6 pounds of butter come to at 18 pence a pound? *Ans.* 108 *pence.*
6. What will 27 pounds of sugar come to at 11 cents a pound? *Ans.* 297 *cents.*
7. At 10 cents a yard, what will 29 yards of cotton cloth come to? *Ans.* 290 *cents.*
8. A barrel of beer contains 36 gallons, and each gallon 4 quarts; how much, then, will it come to at 12 cents a quart? *Ans.* 1728 *cents.*

Oliver bought 33 books at 49 cents apiece; how much did they come to?

Multiplicand 49 }
Multiplier 33 } FACTORS.

Price of 3 books, $147 = 49 \times 3$

Price of 30 books, $147. = 49 \times 30$

Prod. $1617 = 49 \times 33$

Cents, price of 33 books.

First, we multiply by the 3 (*units*) as in Illus. Art. 33; then by the 3 *tens*, saying 3 times 9 is 27 (*tens*): we set down 7 (the unit figure of *this product*) in *ten's* place, which is, *directly under the* MULTIPLYING FIGURE, and carry 2 to the product of the next figure, saying 3 times 4 is 12, and 2 we had to carry is 14. It now appears, that the product by the 3 tens is the same as that by the 3 units; but there is this difference—the figures in the product by the 3

tens are all *removed one place further to the left*, by which their value is increased TEN TIMES. (See Art. 13.) That this product should be *ten times* that by the 3 units, is obvious, because it is produced by a multiplier *ten times as large* (Art. 12.) And it is also obvious, that the sum of the products, by the 3 tens, and the 3 units, is equal to the product by 33. Hence,

35. *To multiply simple numbers,*

(When the multiplier is more than 12,)

RULE I. Multiply the multiplicand by EACH FIGURE in the multiplier and set down the first figure of each product *directly under the* MULTIPLYING FIGURE.

II. Add the *several products* together, and it will give the ANSWER.

Illus. What will 57 head of cattle come to at 34 dollars a head?

57
34
228
171
1938

It is immaterial, as to the result, which of the factors we multiply by; for $7 \times 4 = 4 \times 7$: It is more convenient, however, to make the lesser factor the multiplier.

N. B. Either the multiplicand or the multiplier is always to be considered as an abstract number (*Art. 3, Note.*) For, in strictness of language, it would be improper to speak of multiplying *yards* by *dollars*, or *dollars* by *yards*. These, and such like expressions may not, however, be wholly improper, if they be understood, not literally, but merely as a kind of convenient phrase, to indicate that the *yards are to be multiplied by a* NUMBER *equal to the number of dollars*, or that the *dollars are to be multiplied by a* NUMBER *equal to the number of yards*.

36. PROOF* I. Cast the 9's out of the multiplicand and set the EXCESS on the *left hand* side of an EX.

II. Do the same with the multiplier, and set the excess on the *right hand* side.

III. Multiply these two excesses together, cast the 9's out of *their* product, and set the excess *above* the EX.

IV. Cast the 9's out of the *total product* and set the excess *below*; then if the excess ABOVE, be equal to the excess BELOW, the work may be supposed right.

Illus. In 38 bushels of apples, each bushel containing 87 apples, how many apples?

$87 = 6$
$38 = 2$
696 12
261 9's out leaves
Ans. $3306 = 3$

3 / 6 X 2 / 3

I. *Multiplicand.* 8 and 7 is 15, 9's out leaves 6; which set on the left hand side.

II. *Multiplier.* 3 and 6 is 11, 9's out leaves 2; which set on the right hand side.

III. "Multiply these two excesses together;" $6 \times 2 = 12$, 9's out leaves 3; which set *above* the EX.

IV. *Product.* 3 and 3 is 6, and 6 is 12; 9's out leaves 3; which set *below* the EX.

NOTE. In adding together the digits, as above, the 9's need not be reckoned.

EXAMPLES IN MULTIPLICATION.

	1		2		3	4	5
Mult.	785	3	98954		5236	31416	80305678
By	69	2 X 6	768	X	1142	4752	586393

* This method of proof depends upon a property of the number 9, which is, that "any number divided by 9, will leave the same remainder as the sum of its digits divided by 9."

Illus. Take the number 465. This separated into its parts, becomes $400 + 60 + 5$. But $400 = 4 \times 99 + 4$; and $60 = 6 \times 9 + 6$. Therefore $465 = (4 \times 99 + 4) + (6 \times 9 + 6) + 5 = 4 \times 99 + 6 \times 9 + 4 + 6 + 5$, and 465 divided by $9 = (4 \times$

37. When ciphers occur between any of the significant figures, NEGLECT *those ciphers, and multiply by the significant figures only.*

REMEMBER ALWAYS to place "the first figure of each product directly under the *multiplying figure.*"

	6
Mult.	3586
By	304
	14344
	10758
	1090144

In the first place we multiply by the 4 (*units,*) and then by the 3 (*hundreds*); hence it is plain, that this last product must be HUNDREDS. We omit multiplying by 0, because 0 times any thing is *nothing.*

7	8
23456	2478
7034	5003
X	X

9. Multiply *nine thousand eight hundred and seventy-six* by *five thousand and nine.* *Ans.* 49466884.

10. What will 354 yards of cloth come to at 107 cents per yard? *Ans.* 37878 *cents.*

11. Multiply *eighty-nine thousand five hundred and six* by *three thousand and fifty-two.* *Ans.* 273172312.

12. Bought 406 tons of hay for 984 cents per ton; what did it come to? *Ans.* 398520 *cents.*

CONTRACTIONS IN MULTIPLICATION.

38. A COMPOSITE NUMBER is a number composed of *two or more other numbers* MULTIPLIED TOGETHER.

And 22, 36, 144, &c. are composite numbers.

Because 22=11×2, 36=6×6, 144=12×12, or 8×6×3, &c.

39. The PARTS of which a *composite number* is composed are called its COMPONENT PARTS.

Thus 11 & 2 are the *component parts* of 22; because 11 × 2 = 22.

And 6 & 6 (or 4 & 9, or 3 & 12,) are the component parts of 36; because 6 × 6 (or 4 × 9, or 3 × 12) = 36. And so on.

40. If we multiply any number by 11, and that product by 2, we shall evidently obtain a product equal to (2 times 11 is) 22 times that number. If there should be *three or more* component parts, it is plain, that we first multiply by *one of them,* that product by *another,* this last product by a *third,* and so on.

For instance, to multiply by 144 (the component parts of which being 8, 6 & 3,) we multiply first by 8, and obtain 8 times the given number; that product by 6, and obtain (6 times 8 is) 48 times the given number; and this last product by 3, and obtain (3 times 48 is) 144 times the given number. This method of proceeding may be applied to *any composite number* whatever. Therefore,

To multiply by a composite number,

RULE. Multiply by its COMPONENT PARTS *continually.*

Illus. What will 15 acres of land come to at 29 dollars per acre?

29 dollars, price of	1 acre.
5 *one of the component parts.*	
145 dollars, price of	5 acres.
3 *the other com. pt.*	3×
Ans. 435 dollars, price of	15 acres.

The component parts of 15, are 5 & 3; 5 × 3 = 15. The price of 5 acres is 29 × 5 = 145 dollars; the price of 15 acres is evidently 3 times as much, or 435 dollars, *Ans.*

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99 + 6 × 9 + 4 + 6 + 5) ÷ 9. But 4 × 99 + 6 × 9 is evidently divisible by 9 without a remainder; therefore the given number 465 divided by 9 will leave the same remainder as the sum of its digits divided by 9.

EXAMPLES.

1. How much will 39 yards of cotton cloth come to at 18 cents a yard?
Ans. 702 cents.

2. Suppose there are 376 pensioners in one state, and each one receives 96 dollars *per annum*;* how much will they all receive? ($12 \times 8 = 96$.)
Ans. 36096 dollars.

3. Suppose a man sows 13728 grains of wheat, and each grain produces 42 other grains; how many grains will he harvest? *Ans.* 576576.

41. Annexing a cipher to the right hand of any number *removes it one place towards the left*, and therefore *increases its value* TEN TIMES: and it is therefore, in effect, *multiplying it by* TEN. Hence as many ciphers as we annex to the right hand of a number, so many times we multiply that number by TEN. Therefore,

When the multiplier is 1, *with any number of ciphers annexed,*

RULE. Annex as many ciphers to the right hand of the MULTIPLICAND.

EXAMPLES.

Mult. by		Product.	Mult. 6 by		Product.
10		50	10		60
100		500	100		600
1000		5000	1000		6000
10000		50000	10000		60000

And so of any other number multiplied by 10, 100, 1000, &c.

42. From the principles already explained, it is obvious, that,

When there are ciphers on the left hand of EITHER *or* BOTH *the factors,* We may obtain the product by observing the following

RULE. NEGLECT THOSE CIPHERS, place the other figures under one another, and multiply them as before directed; and to their product annex as many ciphers as were neglected in BOTH FACTORS.

EXAMPLES.

	1		2	3
Mult.	536	It may be observed, that the number 140 is a *composite number*, composed of 14 and 10;—we first multiply by 14, and then by 10, *by annexing* 0. (See Art. 40.)	523600	72350
By	140		7850	12300
	2144			21705
	536			14470
				7235
Prod.	75040			889905000

4. There are 160 square rods in an acre; how many square rods there in 570 acres? *Ans.* 91200.

5. Bought 130 acres of land at 40 dollars per acre; how much did it come to? *Ans.* 5200 dollars.

6. *Seventy* men paid a certain sum, whereof each man paid 1300 dollars; how much did they all pay? *Ans.* 91000 dollars.

7. There are 60 minutes in an hour, and 24 hours in a day; how many minutes then are there in 1 day? —— 1 week? —— 1 year? (*allowing 7 days to a week, and 52 weeks to the year.* *Ans. to last*, 524160

THEORETICAL QUESTIONS.

What is simple multiplication? What the sign? How is it read? What TERMS are used in multiplication? What is the multiplicand? —— the multiplier? —— product? What are BOTH *the multiplicand and the multiplier* sometimes

* A Latin phrase signifying by the year.

called? What is the RULE *for multiplying simple numbers*, when the multiplier does not exceed 12? What, when the multiplier *exceeds* 12? What is the method of proof? What, the first step? —— the second? —— the third?——the fourth? When ciphers occur between any of the significant figures of the multiplier, how do you proceed? What is a *composite number*? What are the *parts* of a composite number called? How do you multiply by a composite number? How do you proceed when the multiplier is 1, with any number of ciphers annexed? How, when there are ciphers on the right hand side of *either* or *both* the factors?

EXAMPLES FOR PRACTICE.

1. Multiply	5749857	by	78.	Product	448488846.
2.	2653294	by	872.	. . . . .	2313672368.
3.	78965987	by	5893.	. . . . .	465346561391.
4.	562916859	by	490070.	. . . . .	275868665090130.

5. In one mile are 320 rods; how many rods in 57 miles?

6. It is 436 miles from Boston to the city of Washington; how many rods is that? *Ans.* 139520.

7. What will 131 yards of Irish linen come to, at 38 cents per yard? *Ans.* 4978 cents.

8. What cost 784 chests of tea at $69 per chest? *Ans.* $54096.

9. 10700 men take a prize, whereof each man receives $46; what was the sum of money taken? *Ans.* $492200.

10. There are 24 hours in a day;—if a ship sail 7 miles in an hour, how far will she sail in one day? ——— 36 days? ——— 365 days? *Ans. to the last*, 61320 miles.

11. If 46 men can do a piece of work in 60 days, how many will it take to do it in one day? *Ans.* 2760.

12. If 1851 men receive $758 apiece, how many dollars will they all receive? *Ans.* $1403058.

13. Two men depart from the same place, and travel in opposite directions, one at the rate of 27 miles a day, the other 31 miles a day; how far apart will they be at the end of 6 days? *Ans.* 348 miles.

14. A merchant bought 13 pieces of cloth, each piece containing 28 yards, at $6 a yard; how many yards were there, and what the whole cost? *Ans. there were* 364 *yards, and it cost* $2184.

15. If 18 men can do a piece of work in 90 days, how long will it take one man to do the same? *Ans.* 1620 days.

16. If the use of $100, for *one year*, be $6, what will be the use for 2 years? ——— 3 years? ——— 7 years? 15 years? ——— 74 years? *Ans. to the last*, $444.

17. If the use of $1, for one year, be 6 cents, what will be the use of $2? ——— $5? ——— $7? ——— $15? ——— $25? (*for the same time.*) *Ans. to the last*, 150 cents.

18. An army has 150 wagons loaded with beef; each has 6 barrels, each barrel contains 2 hundred pounds, and each hundred pounds is worth $3; What is the value of the whole? *Ans.* $5400.

19. What sum of money must be divided among 27 men, that each may receive $115? *Ans.* $3105.

20. If a carriage wheel turn round 346 times in running 1 mile, how many times will it turn round in the distance from New York to Philadelphia, it being 95 miles? *Ans.* 32870.

21. Two men, A and B, start from the same place at the same time, and travel the same way; A travels 52 miles a day, and B 44; how far apart will they be at the end of 14 days? *Ans.* 112 miles.

22. There is a field full well I know,
Wherein just forty trees doth grow—
And on each tree seven boughs there be,
And on each bough seven nests, you see,
And in each nest are blackbirds three;—
Each blackbird eateth in a year,
A bushel of corn or very near;
Then nicely reckon with your pen,
What they will eat in years ten. *Ans.* 58800 bushels.

DIVISION OF SIMPLE NUMBERS,

COMMONLY CALLED

SIMPLE DIVISION.

43. SIMPLE DIVISION is finding how often *one* simple number is contained in *another*, and is a short method of performing SEVERAL SUBTRACTIONS.

Illus. Master James divided 12 pears among 3 of his companions; how many pears did each one receive?

By division 3)12(4 times.
That is, 3 will go in 12, 4 times; therefore each boy will receive 4 pears. *Ans.*

By Subtractions.

12 9 6 3
3 3 3 3
9 6 3 0

Hence it appears that questions in division can be performed by several subtractions. But when the numbers are large the operations become tedious. The *number of subtractions* indicate the quotient.

TABLE.

How many times	in	in	in	in	in	in	in	in	in	in	in
2	4!	6!	8!	10!	12!	14!	16!	18!	20!	22!	24!
3	6!	9!	12!	15!	18!	21!	24!	27!	30!	33!	36!
4	8!	12!	16!	20!	24!	28!	32!	36!	40!	44!	48!
5	10!	15!	20!	25!	30!	35!	40!	45!	50!	55!	60!
6	12!	18!	24!	30!	36!	42!	48!	54!	60!	66!	72!
7	14!	21!	28!	35!	42!	49!	56!	63!	70!	77!	84!
8	16!	24!	32!	40!	48!	56!	64!	72!	80!	88!	96!
9	18!	27!	36!	45!	54!	63!	72!	81!	90!	99!	108!
10	20!	34!	40!	50!	60!	70!	80!	90!	100!	110!	120!
11	22!	33!	44!	55!	66!	77!	88!	99!	110!	121!	132!
12	24!	36!	48!	60!	72!	84!	96!	108!	120!	132!	144!

44. This sign ÷ signifies DIVISION, and is read *divided by*.

As $15 \div 5 = 3$, that is to say, 15 *divided by* 5 is equal to 3.

45. The VINCULUM $\overline{\quad}$ indicates that the *numbers* or *quantities* over which it is placed, *should* FIRST *be united*.

As $15 \times \overline{3+6}$, indicates that 3 & 6 are to be *united*, previous to their being multiplied into 15: hence $15 \times \overline{3+6} = 135$. Here, if it had not been for the vinculum, we should have multiplied 15 by 3, and added 6 to the product, and thus we should have obtained *only* 51.

46. There are FOUR terms to be noticed in division, the *dividend, divisor, quotient*, and *remainder*.

I. The DIVIDEND, is the number given *to be divided.*

II. The DIVISOR is the number given *to divide by.*

III. The QUOTIENT is the number of times the *dividend* contains the *divisor.*

IV. The REMAINDER is *what is left* of the *dividend*, after containing the divisor as many times as is possible.

From this last definition it is evident

1st, That the remainder will always be of the *same name* as the dividend, because it is "what is left" of it; that is, it is *a part of it.*

2dly, That the remainder must always be *less* than the divisor, else it would be "possible" for the dividend to contain the divisor *another time.*

Illus. If 13 men have equal shares in a cotton manufactory, and a dividend of 3579 dollars is to be made out between them; how much will each man receive?

Here it is evident that if we divide 3579 dollars into 13 equal parts, *one* of these parts will be the sum which each man is to receive. Therefore,

Divis. Dividend. Quot.

13)3579*(275
26
—
97
91
—
69
65
—
Remainder 4 *dols.*

1st, We SEEK how often 13 will go into 35 (this being a sufficient number of the left hand figures of the dividend) which we find to be 2 times; therefore we put 2 in the quotient.

2dly, We MULTIPLY the divisor by this quotient figure, and set the product under that part of the dividend used, viz. 35 (00).

3dly, We SUBTRACT the product therefrom, (26 from 35.)

4thly, We BRING DOWN the next figure of the dividend (7) to the right hand of the remainder (9).

Then SEEK, MULTIPLY, SUBTRACT, and BRING DOWN, as before. Hence,

47. *To divide* SIMPLE NUMBERS, we have this

RULE. I. SEEK how often the divisor will go in as many of the left hand figures of the dividend as are just necessary, and set the result in the quotient.

II. MULTIPLY the divisor by this quotient figure, and set the product under that part of the dividend used.

III. SUBTRACT it therefrom.

IV. BRING DOWN the next figure of the dividend to the right hand of the remainder, and proceed as before.

NOTE. If, after having brought down a figure to the remainder, it should *still be less* than the divisor, place a cipher in the quotient, and bring down another figure.

[The foregoing method of performing the operation is called LONG DIVISION.]

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* Analysis.

13)3579(200 Quot. of 2600 ÷ 13
13 × 200 = 2600
—
13) 979(70 Quot. of 910 ÷ 13
13 × 70 = 910
—
13) 69(5 Quot. of 65 ÷ 13
13 × 5 = 65
—
— 275 Quot. of (3579 — 4) ÷ 13.
4 remains undivided = 4
—
Whole dividend 3579

1.
17)4921(289.
34+
—
152
136+
—
161
153+
—
8+
—

4921 PROOF.

PROOF.*

48. ADD the *remainder* and all the *subtrahends* together, and, if THEIR SUM is equal to the DIVIDEND, the work is right.

It will be best to mark the numbers to be added with +, as per examples. Be cautious not to add in any of the upper numbers.

2.
23)7072(307.
69+
—
172
161+
—
11+
—

PROOF 7072

Charles gave 3 boys 15 lemons, how many did he give each one?
15 ÷ 3 how many? *Ans.* 5.
Charles gave 3 boys 5 lemons each, how many did he give all three?
3 × 5 how many? *Ans.* 15.
A man sold 6 swine for 18 dollars, how much was that apiece?
18 ÷ 6 how many? *Ans.* 3 (*dollars.*)
A man sold 6 swine for 3 dollars each, how much was that in all?
6 × 3 how many? *Ans.* 18 (*dollars.*)
Hence it appears that

49. *Division* is the REVERSE of *multiplication*, and may be proved by it.

And indeed, the method we have given, although generally styled the *proof by addition*, is, in reality, *proof by multiplication*, as plainly appears from our illustration of it.

EXAMPLES IN DIVISION.

1. *Nineteen* men drew a prize of 17576 dollars; how much was that apiece? *Ans.* 925 *dollars, and* 1 *dollar left.*
2. Divide 2408881 by 413. *Quot.* 5832, *Rem.* 265.
3. I would plant 2070 fruit trees in 14 rows; how many trees must I place in each row? *Ans.* 155.
4. *Ninety thousand five hundred and twenty-five* dollars are to be raised by *seventy-one* men; how much is that per man? *Ans.* 1275 *dollars.*
5. A paymaster received 153033 dollars to be paid out among 87 men; how much must each man receive? *Ans.* 1759 *dollars.*
6. What number multiplied by 57 will produce the same as 143 multiplied by 71, plus 50? *Ans.* 179.

---

* The several methods of proof may be found in Pike's large Arithmetic, or in Hutton's Mathematics. The one here given is the most practical and may be thus illustrated:

Divisor. Dividend - - - - - = Product, &c.
367) 168696 (459 = 165600 + 1377 + 1719
Divisor × 4 (00) = 1468.. = - - - - - = 1468 - - - - - 400 × Multiplicand
.2189. - - - - -

7. How many times will 517 go in 30380471? *Ans.* 58763.

8. I would fence in a piece of ground for the purpose of setting out 3045 choice fruit trees, so as to have 35 rows, and the trees 33 feet apart; how large a piece must I enclose? *Ans.* 2871 *feet long, by* 1155 *feet wide.*

Divis.	Divid.	Quot.	Multiplying the dividend by 2 we have	1 Divis.	1 Divid. × 2.	1 Quot. × 2
4)	16 (	4		4)	32 (	8

1 Divis. 1 Divid. ÷ 2. Quot. ÷ 2

Dividing the dividend by 2, we have 4) 8 (2 Therefore,

If the DIVISOR *remain the* SAME,

50. *Multiplying* the DIVIDEND by any number is multiplying the QUOTIENT by that number, and *dividing* the DIVIDEND is dividing the QUOTIENT.

For, the *larger* the dividend the OFTENER the *same divisor* will be contained in it; and the *smaller* the FEWER TIMES.

Divis.	Divid.	Quot.	Multiplying the divisor by 2 we have	1 Divis. × 2.	1 Divid.	1 Quot. ÷ 2.
4)	16 (	4		8)	16 (	2

1 Divisor ÷ 2. 1 Divid. 1 Quot. × 2

Dividing the divisor by 2, we have $\frac{2}{2}$) 16 (8 Therefore,

If the DIVIDEND *remain the* SAME,

51. *Multiplying* the DIVISOR by any number, is dividing the QUOTIENT by that number, and *dividing* the DIVISOR is multiplying the QUOTIENT.

For, the *larger* the divisor, the FEWER TIMES it will be contained in the *same dividend* and the *smaller* the OFTENER.

Divisor.	Dividend.	Quotient.
4)	16 (	4

Divid. & Divis. 1 Divis. × 2. 1 Divid. × 2. 1 Quot.

Multiplying BOTH 16 and 4 by 2, we have 8) 32 (4

Divid. & Divis. 1 Divis. ÷ 2. 1 Divid. ÷ 2. 1 Quot.

Dividing BOTH 16 and 4 by 2, we have 2) 8 (4 Therefore,

52. If the *dividend* and *divisor* be BOTH multiplied or BOTH divided by the *same number*, the quotient will NOT BE ALTERED.

This is an evident consequence of the combination of the two preceding articles; in which the effect of the multiplication and division by the same number evidently destroy each other. See Art. 30 *c.*

CONTRACTIONS IN DIVISION.

53. *When the divisor does not exceed* 12, a shorter way is generally made use of, called SHORT DIVISION. It consists in performing the operations in the mind, setting down the quotient figures directly under the respective figures of the dividend, and carrying the remainder to the right.

EXAMPLE. Divide the number 357 by 2.

2)357		First, we consider how often the divisor 2, will go in 3;
	— Rem.	which we find to be *once*, and that 1 remains. We set the
Quot.	178—1	quotient figure 1 underneath, and conceive* the remainder 1,

* Conceiving the remainder to be prefixed to the next figure of the dividend is only giving to it its proper value, or supposing it placed where it actually belongs; for, it is very evident, from our notation, that *every unit of any place or order is equal to* TEN *units of the next right hand place*, and the only way to express the remainder in connection with the next figure of the dividend, is, to prefix it to that next figure. This may be more easily seen from the annexed analysis: by which, we also see, that the first quotient figure [1] is not barely 1, but 100; and that the second quotient figure [7] is not barely 7, but 70; as may likewise be seen from what we have before observed. See Note to Illus. Art. 46, *bottom of the page.*

2)200(100	
2)140(70	Rem.
2) 17(8	
— —	
2)357(178	& 1

to be prefixed to the next figure (5) of the dividend. We then consider how often the divisor will go in the sum (15) that it makes; which we find to be 7 times and that 1 remains. We set the 7 underneath, and conceive the 1 to be prefixed to the next figure (7); and again consider how often the divisor will go in the sum (17;) which we find to be 8 times and that 1 remains. This being the last remainder we place it to the right hand.

Hence, *to perform* SHORT DIVISION, *we have the following*

RULE. *Consider how often the divisor will go in the* FIRST FIGURE *or* FIGURES *of the dividend, set the quotient underneath, and conceive the remainder to be prefixed to the* NEXT *figure; then consider how often the divisor will go in the* SUM THAT IT MAKES, *and proceed as before.*

EXAMPLES.

1. 3)7259 2. 4)8350 3. 5)98762 4. 6)12375 5. 7)34573 6. 8)83987

PROOF. Multiply the *quotient* by the *divisor* and add in the *remainder*.

54. When the divisor is a *composite number*, it is evident, that we may divide, *by dividing by* ONE *of its component parts, that quotient by* ANOTHER, *and so on*. For, as in multiplication, we have made the process of multiplying by the component parts, evident, in obtaining the *product* by a composite number, so here the contrary evidently takes place.

If 15 acres of land cost 435 dollars; what is it per acre? $15 = 3 \times 5$.

3)435 *dollars*, whole cost. (*See Ill.* to *Rule*, Art. 40.)

5)145 *dollars*, price per acre, if there had been only 3 acres.

5 times } For, it is

29 *dollars*, price per acre, there being 15 acres

evident, that the *more* land for the *same money*, the less will be the price PER ACRE, and *vice versa*.*

Hence, *to divide by a* COMPOSITE NUMBER,

RULE. Divide by its COMPONENT PARTS *continually*.

EXAMPLES.

1. Divide 713 dollars among 18 men? $3 \times 6 = 18$. Therefore,

3)713 *dollars.*
6)237 — 2
39 — 3
× 6 + 3
237
× 3 + 2
713 *dollars.*

Proof.

Let the whole sum be divided among 3 of the men, and we obtain a quotient of 237 dollars for each man and have 2 dollars left. Also, let the whole number of men be divided into 3 classes, and let one of the THREE MEN who now hold the money, be at the head of each class. Again, let *each* of these THREE MEN divide the money among his fellows, and the result in each class, it will be seen, will be 39 dollars to each man;—and there will in EACH CLASS also be 3 dollars left. Hence, in the *divisions* by the THREE MEN, there are *three* remainders of 3 dollars each; or (3 times 3 is) 9 dollars remainder; which added to the remainder (2) of the former division, gives for the whole, and *true remainder* 11. This process of finding the TRUE REMAINDER, will, by trial, be seen to apply equally well in the division by numbers consisting of *more than two* component parts. Wherefore we have this

55. RULE. *Multiply the* LAST REMAINDER *by the divisor of the* FORMER

* And the *contrary*. Literally, VICE signifies *a state or condition*, and VERSA *inverted*.

DIVISION, *and add in the remainder of* THAT DIVISION; *take* THIS SUM *as* THEN BEING THE REMAINDER, *and proceed as before.*

2. Divide 8623 by 112. (= 2 × 7 × 8.)

2)8623

7)4311 — 1

8) 615 — 6

76 — 7

7 × 7 + 6 = 55;
then 55 × 2 + 1
= 111, the *true remainder.*

3. Divide 17951 by 72. *Quot.* 249, *True Rem.* 23.
4. Divide 9599 by 96. *Quot.* 99, *True Rem.* 95.
5. Divide 48311 by 132. (= 2 × 6 × 11.) *Quot.* 365, *True Rem.* 131.
6. Divide 60839 by 120. (= 2 × 6 × 10.) *Quot.* 506, *True Rem.* 119.

56. We have seen, Art. 41, that *annexing* a cipher to the right hand of any number INCREASES its value *ten times*, or is, in effect, *multiplying it by* TEN. Whence, it is evident, that *cutting off* a cipher from the right hand of any number DECREASES its value also *ten times*; or is, in effect, *dividing it by* TEN. Hence, as many ciphers as we cut off from the right hand of any number so MANY TIMES *we divide that number by* TEN.

If we cut off *one* cipher, we divide the number by *one* TEN, or simply by TEN; if *two* ciphers by *ten tens*; or 100; if *three* ciphers by 1000, &c. And it is obvious, from our notation, that cutting off *any other figures* will have the same effect; only the figures cut off, must be considered as *some part of the dividend undivided.* Therefore,

When the divisor is 1, *with any number of ciphers annexed,*

RULE. Cut off as many figures from the right hand of the dividend; the figures on the LEFT will constitute the *quotient*, and those on the RIGHT, the *remainder.*

EXAMPLES.

By	Divide	Quo.	Rem.
10	78539	7853	9
100	78539	785	39
1000	78539	78	1539
10000	78539	7	18539

} And so of any other number divided by 10, 100, 1000, &c.

5. In one dollar are 100 cents; how many dollars in 285 cents?
Of what name is this remainder? *Ans.* 2, & 85 *rem.*
(*See* 1*st. inference to definition* IV. *Art.* 46.)
6. How many dollars in 1728 cents? *Ans.* 17, & 28 *cents over.*
7. In one dollar are 1000 mills; how many dollars in 4785 mills?
Ans. 4, & 785 *rem.*
Of what name is the remainder? (*See inference above referred to.*)
8. How many dollars in 37625 mills? *Ans.* 37, & 625 *mills over.*

57. Any number with ciphers annexed, is a composite number, composed of 1 with *as many* ciphers annexed, and the number denoted by the other figures of it.

Thus, 10500 is a composite number, composed of 100, and the number 105. Hence (Arts. 56, and 47 or 53,) it is evident, that we may divide by any number with ciphers annexed, by dividing first by 10, 100, 1000, &c., (Art. 56,) and then by the number denoted by the other figures of it, (Art. 47 or 53); That is, by the process expressed in the following

RULE. *Cut off the* CIPHERS *from the right hand of the* DIVISOR, *and* AS MANY FIGURES *from the right hand of the* DIVI-

DEND; *and divide the* REMAINING *figures of the dividend, by the* REMAINING *figures of the divisor.*

☞ OBSERVE to bring down the figures cut off from the *dividend* to the *right hand of the remainder.*

EXAMPLES.

1. Divide 99933 dollars among 1300 men.

13|00)9993|33(76 *dollars, Ans.*
91
—
89
78
—
1133 *Rem.*

This process is evident as well from Art. 52, as from the above explanation; for cutting off two places of figures from the *dividend and divisor*, is, in effect, dividing both these terms by the same number (100) .... Therefore "the quotient will not be altered."

2. 71000)6781539 — Rem, Quot. 96 — 6539

3. 111|00)4531|98 — Rem. Quot. 41 — 298

4. 3651|00)9438|75(— Rem. Quot. 25 — 31375.

5. How many lots of land may be bought for 18597 dollars, at 150 dollars a lot? *Ans. 523 lots, and there will be* 147 *dollars left.*

58. I. Divide 1 apple among 5 boys.

Divisor. Dividend.
5) 1 (

As the operation now stands, it is plain, that the division cannot be executed. But the quotient may be expressed by $1 \div 5$, or more properly, by $\frac{1}{5}$; which is read *one-fifth*. The division may be performed, however, by dividing the apple, (or conceiving it to be divided,) into 5 *equal parts*. This would give to *each* boy ONE of the five parts, or $\frac{1}{5}$ of the apple, *Ans.*

II. The expressions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., are called FRACTIONS, that is *broken numbers*, or some *part or parts* of the UNIT or WHOLE of any thing. Fractions, it will be seen, ARISE FROM DIVISION, and are proper and adequate expressions for the *quotient.*

III. The *lower number*, which is the DIVISOR in division, is called the *denominator*; and the *upper number*, which is the DIVIDEND, the *numerator.*

IV. Divide 16 apples equally among 5 boys.

Divisor. Dividend. Quotient.
5) 16 ($3\frac{1}{5}$
15
—
Remainder 1 *apple.*

Here dividing as usual, we obtain 3 whole apples for each boy; but there is one apple left, *undivided.* In order, therefore, to divide *all* the apples equally among the boys, we must evidently divide this ONE remaining apple into 5 *equal parts*, and give *one of the parts* to each boy. This will give to each 3 apples, and *one-fifth* of another apple, which is written, $3\frac{1}{5}$ apples, *Ans.*

Hence we observe this important principle, viz., that,

59. The REMAINDER, after division, may be placed over the DIVISOR, and annexed to the quotient.

NOTE. In division, the QUOTIENT does not include the *remainder*, but hereafter the term will be used to express the *remainder*, TOGETHER *with the* OTHER *quotient.*

60. AN INTEGER is the WHOLE of *any thing*, or it is a WHOLE NUMBER, as distinguished from a FRACTION.

Thus, in the above result, the 3 are *integers*, and the $\frac{1}{5}$ a *fraction.*

Numbers consisting of *integers* and *fractions*, as $3\frac{1}{5}$, $2\frac{1}{3}$, &c., are called MIXED NUMBERS.

THEORETICAL QUESTIONS.

What is simple division? What the sign? How is it read? What does the *vinculum* indicate? What terms are to be noticed in division? What is the dividend? —— divisor? —— quotient? —— remainder? What the *first inference* to last? What the *second*? What is the first part of the RULE? What the second? —— the third? —— the fourth? What is this method of performing the operation called? What is the method of proof? What appears from the illustration of this method? (*See Note, bottom of page.*) What is division the *reverse* of? If the DIVISOR remain the *same*, what effect will *multiplying* the DIVIDEND have on the *quotient*? —— *dividing* the DIVIDEND? Why? If the DIVIDEND remain the *same*, what effect will *multiplying* the DIVISOR have on the *quotient*? —— *dividing* the DIVISOR? Why? What if the *dividend* and *divisor* be BOTH multiplied, or BOTH divided, by the *same number*? What if the divisor does not exceed 12? In what does *Short Division* consist? What is the rule? How do you divide by a *composite number*? How do you find the *true remainder*? When the divisor is 1, *with ciphers annexed*, how do you proceed? How do you divide when the divisor is ANY NUMBER *with ciphers annexed*? What are the expressions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. called? What are fractions? Whence do they arise? What is the *lower number*? What the *upper*? What may be done with the *remainder* in division? What is an *integer*? What are numbers consisting of *integers and fractions* called?

EXAMPLES FOR PRACTICE.

1. Divide	296876234	by	64.	Quot.	4638691	Rem.	10.
2.	47989536925	..	735.	.....	65291886	.....	715.
3.	26574983184	..	8962.	.....	2965296	.....	432.
4.	53479689236	..	7684.	.....	6959876	.....	2052.

5. The sum of $1987 is to be divided among 17 men; what will each man receive? *Ans.* 116 *dollars, and* 15 *dollars rem.*

6. A man travelled 663 miles in 17 days; how much was that per day? *Ans.* 39 *miles.*

7. An army of 615 men is provided with 20910 pounds of bread, for 17 days; how much is that per man, for the *whole time*? —— how much for 1 day? *Ans. for the whole time* 34 *pounds, for* 1 *day* 2 *pounds.*

8. A merchant gained 7500 dollars in 6 years; how much was that per day, there being 365 days in a year? *Ans.* 3 *dollars, and* 930 *dollars over.*

9. A man wishes to travel 585 miles in 13 days; how far must he travel each day? *Ans.* 45 *miles.*

10. The distance from the earth to the sun is 95,000,000 of miles, and light moves that distance in about 8 minutes; at what rate does it travel per minute? *Ans.* 11,875,000 *miles.*

11. I have 750 pieces of cloth, and can put no more than 15 pieces in a bale; how many bales will it take? *Ans.* 50.

12. What number must I multiply by 135, that the product may be 505710? *Ans.* 3746.

13. $639,509,421 are to be divided among 4009 men; what will each man receive? *Ans.* $159,269.

14. A man would put 17,484 pounds of tea in 186 chests, how many pounds must he put in each chest? *Ans.* 94.

15. If you distribute $48,750 among a number of men, in such a manner as to give to each one $975, how many men receive a share? *Ans.* 50.

16. A man has an annual income of $3555; how much may he spend per day, to lay up at the year's end $270? *Ans.* $9.

17. If 2760 men can do a piece of work in one day, how many days will it take 46 men to do the same? How many men will do the work in 5 days? —— in 15 days? —— in 120 days? *Ans. to the last,* 23

18. How long will it take to travel 986 miles, at the rate of 34 miles a day? *Ans.* 29 *days.*

19. If a carriage wheel turns round 32870 times in running from New-York to Philadelphia, a distance of 95 miles, how many times does it turn in running 1 mile? *Ans.* 346.

20. Suppose an orchard of 375 trees, each tree bearing on an average 891 apples; and suppose 99 of these apples make a bushel, and 8 bushels, 1 barrel of cider; how many barrels of cider may be made from such an orchard? *Ans.* 421 *barrels, and* 693 *apples over.*

MISCELLANEOUS QUESTIONS.

1. Bought a horse for 137 dollars, and harness for 73 dollars; what did they both cost?

2. If a horse and harness cost 210 dollars, and the harness cost 73 dollars, what is the cost of the horse?

3. There are two numbers, the greater is 318, and the less 282; what is their sum?

4. The sum of two numbers is 600, and the less is 282; what is the greater?

5. Of two numbers, the *greater* is 513, and the *less* 375; what is their *difference?*

6. If the minuend be 5451, and the subtrahend 4515, what is the remainder? If the remainder be 936, and the subtrahend 4515, what is the minuend?

7. The MINUEND and SUBTRAHEND being given, how do you find the REMAINDER? The remainder and subtrahend being given, how do you find the minuend?

8. The SUM of two numbers and ONE of them being given, how do you find the OTHER?

9. The GREATER of two numbers, and their DIFFERENCE being given, how do you find the LESS? The LESSER of two numbers, and their DIFFERENCE being given, how do you find the GREATER?

10. The *sum* of two numbers is 52, and *one* of them is 18; what is the other?

11. The *greater* of two numbers is 34, and their *difference* 16; what is the *less?*

12. The *lesser* of two numbers is 18, and their *difference* 16; what is the *greater?*

13. If one pound of butter cost 15 cents, what cost 17 pounds?
Ans. 255 *cents.*

14. If one bushel of wheat cost 2 dollars, what cost 56 bushels?
Ans. 112 *dollars.*

15. When the price of AN UNIT of any thing is given, how do you find the price of ANY NUMBER of units?

16. If the price of *one gallon* is 3 shillings, what is the price of 18 gallons?
Ans. 54 *shillings.*

17. If the price of *one* yard is 9 pence, what is the price of 13 yards?
Ans. 117 *pence.*

18. If the price of the UNIT be in CENTS, in what will the price of any number of units be? If in DOLLARS, what? If in SHILLINGS, what? If in PENCE, what?

19. When the price of ANY GIVEN NUMBER of units is given, how do you find the price of ONE? In what kind of money will the answer be?

20. If the price of 19 bushels of rye be 1064 cents, what will be the price of *one* bushel? *Ans.* 56 *cts.*

21. If 378 pounds of tea cost 1512 shillings, what is the price of one pound? *Ans.* 4 *shillings.*

The FACTORS being given, how do you find the PRODUCT?

PRODUCT and ONE of the factors being given, how do you find

24. The DIVISOR and QUOTIENT being given, how do you find the DIVIDEND?

25. The dividend and quotient being given, how do you find the divisor?

26. What is the product of the factors 26 and 12?

27. What number multiplied by 12, will produce 312?

28. What number multiplied by 26, will produce 312?

29. If the divisor be 12 and quotient 26, what is the dividend?

30. If the dividend be 312, and quotient 26, what is the divisor?

31. If a man save 6 cents a day, how many cents would that be in 1 year, or 365 days? ——how many in 45 years? ——how many dollars would it be? ——how many cows could he buy with the money at 12 dollars each? *Ans. to the last, 82 cows, and 1 dollar 50 cents rem.*

32. A man sold sixteen head of young cattle, at 23 dollars a head, and a yoke of oxen at 75 dollars; in pay, he received a chaise, worth 114 dollars, and the rest in money. How much money did he receive? *Ans. 329 dols*

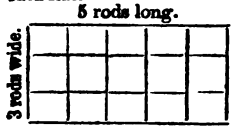
33. Bought 7 pieces of cloth at 63 dollars a piece,
" 385 pr. of shoes at 2 dollars a pair, and
" 514 pieces of calico at 4 dollars a piece,

What is the amount? *Ans.* $3267.

34. The earth, in moving round the sun, travels at the rate of about 68000 miles an hour; how many miles then does it travel in one day, (24 hours?)———how many miles in one year, (or 365 days?)———how many days would it take a man to travel this last distance, at the rate of 40 miles a day?———how many years? *Ans. to last, 40900 years.*

35. There is a piece of ground 5 rods long and 3 wide; how many square rods does it contain?

NOTE. A *square rod* is a square, (like one of those in the figure below,) 1 rod on each side.



Here we see that there are as many squares in each row, as rods on one side, and that the number of rows is equal to the number of rods on the *other* side. It is plain, therefore, that the whole number of square rods in the piece, is obtained by *multiplying the number of rods it is* LONG *by the number of rods it is* WIDE. That is by MULTIPLYING THE LENGTH BY THE BREADTH.

Thus, $5 \times 3 = 15$ *square rods;* (which is called the *area.*) *Ans.*

Definition 1. Two lines are said to be parallel, when they are every where at the same distance from each other.

2. A figure, having its opposite sides equal and parallel, (*like the above,*) is called a *parallelogram*, or *oblong*.

3. A figure having ALL its sides EQUAL AND PARALLEL, (*like one of those in the above,*) is called a SQUARE.*

36. How many square rods in a field 75 rods one way, and 60 the other? *Ans.* 4500 (*square rods.*)

37. In *one acre* are 160 square rods; how many acres in the above mentioned field? *Ans.* 28 (*acres*), *and* 2 *square rods over.*

☞ The *space enclosed* is called the AREA.

38. What is the area of a field 115 rods one way, and 75 the other? *Ans.* 53 *acres and* 145 *rods over.*

39. It is required to find how long a piece of land must be, that is 25 rods wide, to contain 15 acres. *Ans.* 96 *rods.*

40. What is the sum of twice 397, three times 794, four times 3176, five times 15880, six times 95280, and once 333040? *Ans.* ONE MILLION.

41. A cashier received, $409, $2013, $8510, and $928; of which he paid away 15 thousand 15 hundred and 15 dollars; how much has he left?

* THEORETICAL QUESTIONS. What is a square? ——a square rod? When are two lines said to be parallel? What is a parallelogram or oblong? How you find the area of an oblong?

FRACTIONS.

61. A FRACTION is *one or more parts* of a UNIT.

Illus. 4 *farthings* make 1 penny. Here, 1 penny is the UNIT, and 4 *farthings* its *parts*.

1 farthing is *one-fourth* of 1 penny.
3 farthings are *three-fourths* of 1 penny, &c.

62. A VULGAR FRACTION is *one or more parts* of a UNIT, expressed by TWO NUMBERS placed one above the other, with a line between them.

Illus. If 8 *shillings* make 1 *dollar*, then 1 *dollar* is the UNIT, and 8 *shillings* its *parts*.

1 shilling is $\frac{1}{8}$, (*one-eighth*,) of 1 dollar.
3 shillings are $\frac{3}{8}$, (*three-eighths*,) of 1 dollar.
5 shillings are $\frac{5}{8}$, (*five-eighths*,) of 1 dollar, &c.

63. The LOWER NUMBER is called the DENOMINATOR, because it GIVES NAME to the fraction.

When a *unit* is divided into 3 *parts*, the parts are NAMED *thirds*.
When a *unit* is divided into 4 *parts*, the parts are NAMED *fourths*.
And so on, the *parts* always taking their NAME from the *number* of them into which a UNIT is divided.

64. The DENOMINATOR represents the *number of parts* into which the UNIT is divided.

If an apple is divided into 4 *parts*, 1 of these parts will be expressed by, $\frac{1}{4}$ *of the apple*. The denominator, 4, representing that the unit, (which is here an apple,) is divided into 4 parts; and the upper number, 1, denoting that 1 part is used.

65. The UPPER NUMBER is called the NUMERATOR, because it shows the *number* of parts used.

Illus. Master Henry divided an apple into 6 equal parts, but one of the parts being unsound, he ate only 5: consequently he ate only $\frac{5}{6}$ of the apple. Here "the *denominator* (6) represents the number of parts into which" the apple was divided, and the *numerator* (5) "shows the number of parts used."

66. The word, TERMS, is used to express BOTH *numerator and denominator of the same fraction.*

As the terms of the fraction, $\frac{7}{12}$, are 7 and 12.

67. A PROPER FRACTION is one whose *numerator* is EQUAL TO, or LESS than its denominator.

As $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$; $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, &c.

68. A MIXED NUMBER is a WHOLE NUMBER *and a* FRACTION.

$1\frac{1}{2}$ is a mixed number, and is read, *one* and a *half*.
$2\frac{1}{3}$ is a mixed number, and is read, *two* and *one-third*, &c.

It is plain that $1\frac{1}{2}$ is equal to 3 *halves*; for 1 is equal to 2 *halves*, and 2 *halves* and 1 *half* is equal to 3 *halves*, or $\frac{3}{2}$.

It is also plain, that $2\frac{1}{3}$ is equal to 7 *thirds*; for 2 is equal to 6 *thirds*, and 6 *thirds* and 1 *third* is equal to 7 *thirds*, or $\frac{7}{3}$.

69. An IMPROPER FRACTION* is one whose *numerator* is GREATER than its *denominator*.

---

* So called on account of its being GREATER than "*a unit*," which is not according to the definition that "a fraction is one or more parts of *a unit*." Now *a unit* if *a fraction* is one or more parts of ONE UNIT, it is evidently fraction GREATER than one unit, except it be under the title

Expressions like these, $\frac{3}{2}$, $\frac{7}{4}$, $\frac{17}{5}$, &c., are improper fractions.

The expression $\frac{3}{2}$, is equal to $1\frac{1}{2}$; for, as 3 halves are equal to $\frac{2}{2}$ and $\frac{1}{2}$, and as $\frac{2}{2}$ are equal to 1, 3 halves must be equal to $1\frac{1}{2}$.

So also the expression $\frac{7}{3}$, is equal to $2\frac{1}{3}$; for, as 7 thirds are equal to $\frac{6}{3}$ and $\frac{1}{3}$, and as $\frac{6}{3}$ are equal to 2, 7 thirds must be equal to $2\frac{1}{3}$.

70. A fraction whose *numerator* is EQUAL to its *denominator* is equal to ONE.

Illus. If a unit is divided into any certain number of equal parts, and if all these parts are used, the whole is used. Hence it is evident,

1st. *That a fraction whose numerator is* LESS *than its denominator, is* LESS *than* ONE.

2d. *That a fraction whose numerator is* GREATER *than its denominator, is* GREATER *than* ONE.

71. REDUCTION of fractions, is the changing of them from *one form to* ANOTHER.

This is done in order to prepare them for certain operations in Arithmetic, by giving to them a *different* FORM, *retaining the same value.*

The *mixed number* $3\frac{5}{7}$ may be reduced to the improper fraction $\frac{26}{7}$, by saying 5 times 3 is 15, and 2 is 17, and then placing 5 under 17, thus, $\frac{17}{5}$. Hence,

72. *To reduce a* MIXED NUMBER *to an* IMPROPER FRACTION,

RULE. Multiply the WHOLE NUMBER by the *denominator* of the fraction, and add in the *numerator;* under this SUM place the *same denominator.*

Illus. Reduce $4\frac{5}{6}$ to an improper fraction.

"Multiply the whole number" 4 $\frac{5}{6}$

"By the denominator of the fraction" 6 "and add in the numerator" 5.

The sum is 29; "under this sum place the same denominator" 6, and the result is $\frac{29}{6}$, the improper fraction required.

In this example we say, 6 times 4 is 24 and 5 is 29. And then we place the denominator 6, under the sum 29, as the rule directs.

EXAMPLES.

1. Reduce $8\frac{7}{9}$ to an improper fraction. *Ans.* $\frac{79}{9}$
2. Reduce $17\frac{5}{7}$ to an improper fraction. *Ans.* $\frac{124}{7}$
3. Reduce $98\frac{9}{11}$ to an improper fraction. *Ans.* $\frac{1087}{11}$
4. Reduce $105\frac{13}{19}$ to an improper fraction. *Ans.* $\frac{2008}{19}$

73. *To reduce a* WHOLE NUMBER *to the* FORM *of a fraction having a* GIVEN DENOMINATOR,

RULE. Multiply the WHOLE NUMBER by the *given denominator,* and under the product place the SAME denominator.

EXAMPLES.

1. Reduce 2 to the form of a fraction having 1 for its denominator.

Any number whatever may be reduced to the form of a fraction by placing under it for a *denominator.*

2. Reduce 5 to the denominator 4. *Ans.* $\frac{20}{4}$

3. Reduce 7 to the denominator 15. *Ans.* $\frac{105}{15}$

74. The VALUE of a fraction is the QUOTIENT of the *numerator* divided by the *denominator*.

The *value* of $\frac{7}{3}$ is $2\frac{1}{3}$; because $2\frac{1}{3}$ is the quotient of $7 \div 3$.

The *value* of $\frac{12}{4}$ is 3; because 3 is the quotient of $12 \div 4$.

To reduce an improper fraction to its value, it is evident that, the preceding operations must be *reversed*.

The improper fraction $\frac{17}{5}$ may be reduced to its value $3\frac{2}{5}$, by dividing 17 by 5, and placing the remainder over the divisor. Thus, 5) 17 ($3\frac{2}{5}$. Hence,

75. *To reduce an* IMPROPER FRACTION *to its* VALUE, (that is, to its equivalent whole or mixed number,)

RULE. Divide the numerator by the denominator, and if there is a remainder, place it over the divisor and annex it to the quotient.

1. Reduce $\frac{12}{4}$ to its value

4) 1 2 (3 *Ans.*
1 2
0

Here, as there is no remainder, the value of the fraction is a *whole number*.

2. Reduce $\frac{29}{6}$ to its value.

6) 2 9 ($4\frac{5}{6}$ *Ans.*
2 4
5 Remainder.

Here, as there is a remainder, the value of the fraction is a *mixed number*.

EXAMPLES.

1. Reduce $\frac{70}{9}$ to its value. *Ans.* $8\frac{7}{9}$
2. Reduce $\frac{124}{7}$ to its value. $17\frac{5}{7}$
3. Reduce $\frac{3000}{20}$ to its value. 150
4. Reduce $\frac{139}{3}$ to its value. $69\frac{2}{3}$
5. Reduce $\frac{6827}{100}$ to its value. $98\frac{27}{100}$
6. Reduce $\frac{2000}{19}$ to its value. $105\frac{5}{19}$

The value of $\frac{16}{4}$ is 4. *Multiplying* the numerator 16 by 2, we have $\frac{32}{4} = 8$; equal to the value of the original fraction *multiplied* by 2. *Dividing* the numerator 16 by 2, we have $\frac{8}{4} = 2$, equal to the value of the original fraction *divided* by 2. Therefore,

If the DENOMINATOR *of a fraction remain the* SAME,

76. *Multiplying* the NUMERATOR by any number, is multiplying the VALUE by that number, and *dividing* the *numerator* is dividing the VALUE.

FOR, by multiplying the numerator, we multiply the "number of parts used" (65), and by dividing the numerator, we divide the "number of parts used" (65); therefore, if the denominator remain the same, the VALUE of the fraction will, in the first case, be MULTIPLIED, and in the second DIVIDED.

The value of $\frac{16}{4}$ is 4. *Multiplying* the denominator 4 by 2, we have $\frac{16}{8} = 2$; equal to the value of the original fraction *divided* by 2. *Dividing* the denominator 4 by 2, we have $\frac{16}{2} = 8$; equal to the value of the original fraction *multiplied* by 2. Therefore,

If the NUMERATOR *remain the* SAME,

77. *Multiplying* the DENOMINATOR by any number is dividing by that number, and dividing the DENOMINATOR MULTIPLYING the VALUE.

For, multiplying the denominator, would be dividing the unit *into more parts*, (63 and 64); therefore, if the same number of parts be used (65), the value of the fraction will be *divided*, because THE PARTS will be AS MANY TIMES SMALLER: And, dividing the denominator, would be dividing the unit into *less parts* (64), therefore if the same number of them be used (65,) the value of the fraction will be *multiplied*, because THE PARTS will be AS MANY TIMES LARGER.

The value of $\frac{16}{4}$ is 4, *Multiplying* both NUM. and DENOM. by 2, we have $\frac{32}{8} = 4$; equal to the value of the original fraction, *precisely*. *Dividing* both NUM. and DENOM. by 2, we have $\frac{8}{2} = 4$; equal to the value of the original fraction. Hence,

78. If the *numerator* and *denominator* be BOTH multiplied or BOTH divided by the *same number*, the VALUE of the fraction will NOT BE ALTERED.

THIS is an evident consequence of the combination of the two foregoing articles, in which the multiplication and division by the same number exactly compensate each other. See Art. 52.

79. *Multiplying* by a FRACTION is taking a PART of the multiplicand as many times as there are LIKE PARTS of a UNIT in the multiplier.

Now the denominator of a fraction "represents the number of parts into which the unit is divided;" and the numerator "shows the number of parts used." (Art. 64 and 65.) Therefore, in multiplying by a fraction, the multiplicand is to be divided into such parts as are represented by the denominator, and then one of these parts is to be used or repeated as many times as there are like parts of a unit in the multiplier.

Multiplying 12 by $\frac{3}{4}$, is taking the 4th part of 12 3 *times*.

That is dividing 12 by 4, and using or repeating the quotient 3 times.

Hence the most rational method of multiplying by a fraction is

TO DIVIDE *the multiplicand by the* DENOMINATOR *of the fraction, and* MULTIPLY *the quotient by the* NUMERATOR.

EXAMPLE. Suppose 5 is to be multiplied by $\frac{3}{4}$.

4) 5
—
Art. 59. $1\frac{1}{4}$
3
—
Prod. $3\frac{3}{4}$

In this operation, it is required to multiply the mixed number $1\frac{1}{4}$ by 3; but as the learner is here supposed to be unacquainted with multiplying a *mixed number*, it would be improper, in this place, to introduce a method involving such a multiplication. The difficulty, however, may be avoided, by reversing the order of the rule. Because it makes no difference whether we first divide 5 by 4, and multiply the quotient by 3, or first multiply 5 by 3, and divide the product by 4; for the result in either case, will be $3\frac{3}{4}$. Hence,

80. *To* MULTIPLY *by a* FRACTION,

RULE. Multiply by the NUMERATOR and divide by the DENOMINATOR.

Illus. Multiply 5 by $\frac{3}{4}$.

5
3
—
4)15
—
$3\frac{3}{4}$

NOTE 1. If the multiplier be ONE, the product will be *equal* to the multiplicand. $6 \times 1 = 6$, $7 \times 1 = 7$, &c.

2. If the multiplier be MORE than ONE, the product will be *more* than the multiplicand. $6 \times 1\frac{1}{2} = 9$, $7 \times 1\frac{1}{2} = 10\frac{1}{2}$, &c.

3. If the multiplier be LESS than ONE, the product will be *less* than the multiplicand. $6 \times \frac{1}{2} = 3$, $7 \times \frac{1}{2} = 3\frac{1}{2}$, &c.

4. When the numerator of the fraction is 1, it is evident, that we need only to observe this

RULE. DIVIDE the *multiplicand by the* DENOMINATOR.

EXAMPLES.

1. Multiply 89 by $\frac{7}{8}$.	*Ans.* $77\frac{7}{8}$	5. Multiply 115 by $\frac{4}{11}$.	*Ans.* $41\frac{4}{11}$
2. Multiply 160 by $\frac{1}{2}$.	80	6. Multiply 171 by $\frac{1}{9}$.	$24\frac{3}{9}$
3. Multiply 225 by $\frac{3}{5}$.	135	7. Multiply 308 by $\frac{5}{13}$.	$118\frac{4}{13}$
4. Multiply 709 by $\frac{1}{20}$.	$27\frac{7}{20}$	8. Multiply 901 by $\frac{17}{19}$.	$806\frac{3}{19}$

81. *To multiply a* FRACTION *by a* WHOLE NUMBER,

RULE. Multiply the *numerator* of the fraction by the multiplier, and the VALUE of the fraction thence arising will be the product required.

NOTE. If more convenient, we may divide the DENOMINATOR, instead of multiplying the numerator, (Art. 77.)

1. Multiply $\frac{3}{4}$ by 8. 8 *times* 3 *is* $\frac{24}{4}$, and the *value* of $\frac{24}{4}$ is 6.
2. Multiply $\frac{7}{4}$ by 2. Denominator $\div$ 2, gives $\frac{7}{2} = 1\frac{3}{2}$

EXAMPLES.

1. Multiply $\frac{3}{4}$ by 4.	*Ans.* $1\frac{3}{4}$	4. Multiply $\frac{5}{6}$ by 10.	*Ans.* $8\frac{3}{3}$
2. Multiply $\frac{3}{7}$ by 12.	$5\frac{1}{7}$	5. Multiply $\frac{107}{112}$ by 56.	$53\frac{1}{2}$
3. Multiply $\frac{27}{40}$ by 7.	$3\frac{6}{7}$	6. Multiply $\frac{1712}{1720}$ by 40.	$39\frac{161}{172}$

82. MULTIPLYING by a *mixed number* is taking the multiplicand *a certain number of times*, and PART of *another* time.

Multiplying 4 by $3\frac{1}{2}$, is taking 4 *three times*, and *one-half of another time.*

Multiplying 7 by $4\frac{1}{3}$, is taking 7 *four times*, and *one-third of another time*, &c. Hence,

83. *To* MULTIPLY *by a* MIXED NUMBER,

RULE. Multiply by the WHOLE NUMBER and then by the FRACTION, and add the latter product to the former.

1. Multiply 212 by $9\frac{5}{7}$.

	212	212
	$9\frac{5}{7}$	5 See Art. 80.
Pro. by the *whole number* (9)	1908	7) 1060
Pro. by the *fraction* $(\frac{5}{7})$	$151\frac{3}{7}$	$151\frac{3}{7}$, *prod. by the fraction.*
Total product	$2059\frac{3}{7}$, *Ans.*	

2. Multiply 75 by $3\frac{1}{4}$

	75
	$3\frac{1}{4}$
Prod. by the whole number (3)	225
Prod. by the fraction $(\frac{1}{4})$	$18\frac{3}{4}$, *obtained by dividing* 75 *by* 4.
Total product	$243\frac{3}{4}$, *Ans.*

Here, to multiply by the fraction, the numerator of which being 1, we have only to divide by the *denominator*. *See Note* 4, *to Art.* 80.

EXAMPLES.

1. Multiply 25 by $5\frac{3}{4}$.	*Ans.* $143\frac{3}{4}$	4. Multiply 754 by $23\frac{5}{6}$.	*Ans.* $17970\frac{5}{6}$
Multiply 96 by 17 s	1672	5. Multiply 785 by $31\frac{9}{18}$.	$24878\frac{9}{18}$
	6900	6. Multiply 855 by $39\frac{1}{3}$.	$33558\frac{1}{3}$

84. *To multiply a* MIXED NUMBER *by a* WHOLE NUMBER,

RULE. Multiply the *integral part* of the multiplicand by the multiplier, then the *fractional part*, (Art. 81,) and add the latter product to the former.

Multiply $95\frac{3}{7}$ by 43.	$95\frac{3}{7}$	43 times $\frac{3}{7}$ is $\frac{129}{7}$, and
	43	the value of $\frac{129}{7}$ is $12\frac{5}{7}$
		See Art. 81; *see also*
Product by the whole numbers (95 by 43)	285	*Arts.* 74, 75, *and* 59.
	380	
Product of the fraction $\frac{3}{7}$ by 43	$12\frac{5}{7}$	
Total product	$4097\frac{5}{7}$	*Ans.*

EXAMPLES.

1. Multiply $256\frac{1}{7}$ by 35. *Ans.* 8965
2. Multiply $72\frac{5}{6}$ by 15. $1092\frac{1}{2}$
3. Multiply $365\frac{1}{4}$ by 50. *Ans.* $18262\frac{1}{2}$
4. Multiply $93\frac{2}{5}$ by 62. $5803\frac{1}{5}$

84 *a. When the multiplier is* SMALL, *it will generally be most convenient to change the order of the foregoing rule in the following manner:*

RULE. First multiply the *fraction* by the multiplier, and then the *whole number*, observing to carry the INTEGERS in the former product to the latter, and to set down the fraction over, (if any.)

Multiply $69\frac{1}{2}$ by 5.	$69\frac{1}{2}$	First, we multiply the fraction $\frac{1}{2}$ by 5, saying 5
	5	times $\frac{1}{2}$ is $\frac{5}{2}$. Then by Art. 74, we find that, in $\frac{5}{2}$
Total product	$347\frac{1}{2}$	there are 2 *integers*, and $\frac{1}{2}$ over. Therefore we set down $\frac{1}{2}$ and carry 2 to the product of the whole

numbers (69 by 5), saying 5 times 9 is 45 and two is 47, set down 7 and carry 4, &c.

EXAMPLES.

1. Multiply $217\frac{1}{2}$ by 3. *Ans.* $652\frac{1}{2}$
2. Multiply $86\frac{3}{5}$ by 4. $346\frac{2}{5}$
3. Multiply $33\frac{1}{4}$ by 3. *Ans.* 100
4. Multiply $567\frac{3}{7}$ by 7. $3974\frac{1}{4}$

85. In *dividing* by a fraction, *the* DIVIDEND *is to be reduced to a denominator equal to that of the* DIVISOR.

In dividing 6 by $\frac{2}{3}$, 6 is to be reduced to the denominator 3; but to reduce it to this denominator, each unit of it must be supposed divided into three parts; and consequently the number that will represent all these parts must be as many times 3 as the number of integral units indicates: If the number of integral units be 6, as above, the number that will represent the *thirds* contained in all these units, must be 18. Because, if 1 unit divided by 3, make 3 parts, 6 units divided in the same way, must make 6 times 3 parts, and 6 times 3 parts, is 18 parts. But if we invert the order of this multiplication, and multiply the 6 (*units*) by the 3 parts, we shall obtain the same product. For $3 \times 6 = 18$, and $6 \times 3 = 18$. As this inverted order is more practical than the other, we shall adopt it.

According, then, to this inversion, the operation of the above example will stand as follows:

6 (*units*) when divided each into 3 parts, (Art. 73,)

3

become 18 thirds, or $\frac{18}{3}$.

And 18 *thirds* are divided by 2 *thirds*, as whole numbers, thus:

2)18(9 the true quotient.
18

NOTE. It may, perhaps, at first be thought strange that the quotient should ever be greater than the dividend. But this will always be the case when the divisor is less than 1. For then the divisor is contained in 1, more than one time, and therefore in *any number of ones*, more than the *same number of times*: $\frac{1}{2}$ is contained in 1, *three times*, and therefore in 2, *six times*, in 3, *nine times*, &c. Hence,

86. *To* DIVIDE *by a* FRACTION,

RULE. ***Multiply*** by the DENOMINATOR, and divide by the NUMERATOR.

Illus. Divide 12 by $\frac{4}{5}$

	12
Multiply by denom.	4
	—
And divide by num.	3)48
	—
Ans.	16

Here the whole number or multiplicand is multiplied by 4, to reduce it to *fourths* of a unit, then these *fourths* are divided by 3, because there are 3 *fourths* of a unit in the divisor.

EXAMPLES.

1. Divide 27 by $\frac{4}{5}$.	*Ans.* $32\frac{3}{4}$	4. Divide 135 by $\frac{3}{5}$.	*Ans.* 225
2. Divide 1075 by $\frac{7}{16}$.	$2457\frac{1}{7}$	5. Divide 72 by $\frac{11}{15}$.	$98\frac{2}{11}$
3. Divide 144 by $\frac{3}{12}$	576	6. Divide 335 by $\frac{17}{16}$.	$374\frac{7}{17}$

87. *To divide a* FRACTION *by a* WHOLE NUMBER,

It is evident, from Arts. 76 and 77, that we have this

RULE. ***Multiply*** the DENOMINATOR or ***divide*** the NUMERATOR, as may be most convenient.

EXAMPLES.

1. Divide $\frac{1}{2}$ by 2. *Denom.* $\times 2 = \frac{1}{2}$ *Ans.* (Art. 77.)
2. Divide $\frac{2}{3}$ by 2. *Num.* $\div 2 = \frac{1}{3}$ *Ans.* (Art. 76.)

3. Divide $\frac{1}{2}$ by 4.	*Ans.* $\frac{1}{12}$	6. Divide $\frac{3}{9}$ by 4.	*Ans.* $\frac{1}{12}$
4. Divide $\frac{2}{15}$ by 3.	$\frac{2}{15}$	7. Divide $\frac{27}{9}$ by 7.	$\frac{3}{7}$
5. Divide $\frac{22}{8}$ by 3.	$\frac{22}{24}$	8. Divide $\frac{64}{96}$ by 8.	$\frac{1}{96}$

88. *To divide by a* MIXED NUMBER,

RULE. Reduce the ***mixed number*** to an ***improper fraction*** (72), THEN DIVIDE as directed in the last article.

Divide 17 by $2\frac{3}{4}$.	$2\frac{3}{4}$	Then, (Art. 87,)	17
(See Art. 72.)	4	Multiplied by denom.	4
	—		—
The improper fraction	$\frac{11}{4}$	And divided by num.	11)68
			—
		Gives	$6\frac{2}{11}$ *Ans.*

NOTE. *It will sometimes be more practical to place the operation in this form:*

$2\frac{3}{4}$)17	
4 4	
— —	
11)68	
—	
$6\frac{2}{11}$	

But this operation is the same as the above, only differently placed. For here the denominator of 11 is *understood* to be 4; and the whole number (17) is multiplied by this denominator, and divided by the numerator (11) as in the above operation.

EXAMPLES.

1. Divide 35 by $4\frac{4}{6}$.	*Ans.* $7\frac{7}{20}$	4. Divide 315 by $17\frac{9}{11}$.	*Ans.* $17\frac{13}{188}$
$8\frac{1}{2}$ by $15\frac{3}{4}$.	$42\frac{21}{110}$	5. Divide 511 by $23\frac{3}{8}$	$21\frac{88}{187}$
by $69\frac{1}{4}$.	$8\frac{138}{150}$	6. Divide 72 by $6\frac{1}{4}$.	$10\frac{8}{11}$

89. *To divide a* MIXED NUMBER *by a* WHOLE NUMBER,

RULE. First divide the INTEGRAL PART of the dividend, and then reduce the remainder (if any) together with the FRACTIONAL PART, to an improper fraction, which divide according to Art. 87.

NOTE 1. If the integral part be *less* than the divisor, the quotient will be 0, and the integral part will itself be the remainder, and must be used as such.

2. If in dividing the integral part, there should be no remainder, then the fractional part must be divided as directed in the article above referred to.

Divide $87\frac{1}{2}$ by 7.

$7)87\frac{1}{2}$

$12\frac{1}{2}$ Ans.

Here, we say 7 is in 8, *once* and 1 over; 7 is in 17, *twice* and 3 over. This remainder, together with the fractional part, gives $3\frac{1}{2}$, which reduced to an improper fraction, becomes $\frac{7}{2}$. (Art. 72.) Then, $\frac{7}{2}$ divided by 7, gives $\frac{1}{2}$. (Art. 87.) *See the operation.*

EXAMPLES.

1. Divide $2\frac{1}{2}$ by 3.	*Ans.* $\frac{5}{6}$	4. Divide $17\frac{1}{2}$ by 2.	*Ans.* $8\frac{3}{4}$
2. Divide $24\frac{3}{4}$ by 3.	$8\frac{1}{4}$	5. Divide $35\frac{2}{5}$ by 3.	$11\frac{4}{5}$
3. Divide $49\frac{1}{7}$ by 5.	$9\frac{2}{35}$	6. Divide $27\frac{1}{2}$ by 9.	$3\frac{7}{72}$

90. *When the dividend and divisor are* BOTH MIXED NUMBERS, *and the denominator of the fractional part of the former is equal to that of the latter,*

RULE. Reduce them BOTH to *improper fractions*, then divide the NUMERATORS.

Divide $192\frac{1}{2}$ by $5\frac{1}{2}$

$5\frac{1}{2})192\frac{1}{2}$

2 2

11)385 *halves*, $35\frac{5}{11}$

9 35 *Ans.*

To illustrate this still further, let it be required to find how often $1\frac{1}{2}$ dollars can be had in $4\frac{1}{2}$ dollars. $1\frac{1}{2}$ dollars = 3 *half dollars*, and $4\frac{1}{2}$ dollars = 9 *half dollars*: Hence it is as obvious, that 3 *half dollars* will go in 9 *half dollars* 3 TIMES, as that 3 will go in 9 *three times*.

EXAMPLES.

1. Divide $23\frac{1}{2}$ by $4\frac{1}{2}$.	*Ans.* 5	3. Divide $1042\frac{1}{2}$ by $69\frac{1}{2}$.	*Ans.* 15
2. Divide $2150\frac{2}{3}$ by $16\frac{1}{3}$.	$132\frac{2}{49}$	4. Divide $987\frac{1}{8}$ by $7\frac{3}{8}$.	$126\frac{1}{47}$

THEORETICAL QUESTIONS.

What is a fraction? What a vulgar fraction? What is the lower number called? Why? What does it represent? What is the upper number called? Why? What is the word *terms* used for? What is a proper fraction? What a mixed number? —— an improper fraction? When the *numerator* of a fraction is EQUAL TO its *denominator*, what is the FRACTION *equal to*? What, when the NUMERATOR is LESS? What, when the numerator is GREATER? What is reduction of fractions? How do you reduce a *mixed number* to an improper fraction? —— a *whole number* to the form of a fraction having a *given denominator*? What the VALUE of a fraction? How do you reduce an *improper fraction* to its VALUE?

If the DENOMINATOR of a fraction remain the *same*, what effect will *multiplying* the NUMERATOR have upon its VALUE? —— *dividing* the NUMERATOR? Why? If the NUMERATOR of a fraction remain the *same*, what effect will *multiplying* the DENOMINATOR have upon its VALUE? —— *dividing* the DENOMINATOR? Why? What if the *numerator* and *denominator* be BOTH multiplied or BOTH divided by the SAME NUMBER? What is multiplying by a *fraction*? How do you *multiply* by a FRACTION? What will the product be, if the multiplier is ONE? What, is *more* than ONE? *What, if less* than ONE? How do you multiply by a fraction

when the numerator is 1? How do you multiply a *fraction* by a *whole number*? What is multiplying by a *mixed number*? How do you multiply by a *mixed number*? How do you multiply a *mixed number* by a *whole number*? How, in this case, do you proceed when the multiplier is SMALL?

In *dividing* by a fraction, what is to be done with the *dividend*? How do you *divide* by a *fraction*? How do you divide a *fraction* by a *whole number*? How do you divide by a *mixed number*? How do you divide a *mixed number* by a *whole number*? How do you divide, when the dividend and divisor are BOTH *mixed numbers* and the denominator of the fractional part of the former is equal to that of the latter?

EXAMPLES FOR PRACTICE.

1. What will 87 barrels of cider come to at $\frac{1}{4}$ of a dollar per barrel? *Ans.* $87\frac{1}{4}$.

2. How much will 131 yards of riband come to at $\frac{1}{8}$ of a dollar per yard? *Ans.* $\$16\frac{3}{8}$.

3. What will $11\frac{3}{4}$ tons of hay come to at $14 per ton? *Ans.* $\$164\frac{1}{2}$.

4. How many yards of carpeting will it take to cover a square room, containing 25 square yards, if the carpeting be $\frac{3}{4}$ yard wide? *Ans.* $33\frac{1}{3}$.

5. If 14 bushels of salt cost $\$9\frac{1}{14}$, what is it per bushel? *Ans.* $\$\frac{127}{126}$.

6. A man sold 11 bushels of grain, for $\$8\frac{1}{4}$; what did he receive per bushel? *Ans.* $\$\frac{33}{44}$.

7. If a man's income be $725 a year, what is it per calendar month? *Ans.* $\$60\frac{5}{12}$.

8. A gentleman owns $\frac{2}{11}$ of a farm worth $1637; what is the value of his part? *Ans.* $\$297\frac{7}{11}$.

9. Bought 13 gallons of gin for $\$9\frac{7}{9}$; what did I give per gallon? *Ans.* $\$\frac{11}{117}$.

10. A butcher bought 8 sheep for $\$13\frac{1}{2}$; what did he give for each one? *Ans.* $\$1\frac{11}{16}$.

11. If one acre of land be worth $\$58\frac{13}{17}$, what will be the value of $\frac{1}{7}$ of an acre?——$\frac{2}{7}$?——$\frac{3}{7}$?——$\frac{4}{7}$?——$\frac{5}{7}$?——$\frac{6}{7}$? *Ans. to the last,* $\$50\frac{44}{119}$.

COMPOUND NUMBERS.

91. DENOMINATION signifies NAME.

In the number 23 *hours*, 14 *minutes* and 27 *seconds*, there are *three denominations*, viz., HOURS, MINUTES, and SECONDS.

92. COMPOUND NUMBERS are numbers of DIFFERENT DENOMINATIONS.

	Pounds, shillings	and	pence;	Feet	and	inches.
As	2 10	&	6;	5	&	4.

There are two *compound numbers*, the first consists of THREE *denominations*, namely, POUNDS, SHILLINGS, and PENCE; and the second of TWO, namely, FEET and INCHES.

The only difference, therefore, between a *simple* and a *compound number*, is, that a *simple number* always consists of the SAME NAME (Art. 19); whereas a *compound number* consists of DIFFERENT NAMES.

REDUCTION.

93. REDUCTION is the CHANGING of numbers from *one denomination* to *another*, without altering their value.

It is of TWO kinds, *Descending* and *Ascending*.

REDUCTION DESCENDING

94. Is the changing of numbers from great denominations to small;

As *pounds* to *shillings*, *feet* to *inches*, &c.

This is performed by MULTIPLICATION.

ILLUSTRATION.

20 12 4

£. s. d. f.

Reduce 5 17 11 3 to farthings.

20

117 *shillings*.

12

1415 *pence*.

4

5663 *farthings*.

Here, to reduce the *pounds* (5) to shillings, we multiply them by 20, and add in the shillings (17)—and thus obtain 117 *shillings*. We then, to reduce the shillings (117) to pence, multiply them by 12, and add in the pence (11)—and thus obtain 1415 *pence*. So we proceed with the pence, (1415), to reduce them to farthings we multiply them by 4, and add in the farthings (3)—and obtain 5663 *farthings*, the denomination required.

That is, we observe the process expressed in the following

RULE.

96. Multiply the *highest denomination* given by so many of the NEXT LESS as make ONE of *that greater*, and add in the next less, if any; and so continue until you obtain the denomination required.

REDUCTION ASCENDING

95. Is the changing of numbers from small denominations to great;

As *shillings* to *pounds*, *inches* to *feet*, &c.

This is performed by DIVISION.

ILLUSTRATION.

Reduce 5663 *far.* to pounds, (and the *less denominations*, if any.)

4)5663 *far.*

12)1415*d.* 3*far.*

20)117*s.* 11*d.* 3*far.*

£5 17*s.* 11*d.* 3*far.*

Here, to reduce the farthings to pence, we divide them by 4, and obtain 1415 *pence* and 3 farthings over. (See 1 Infer. Def. IV. Art. 45.) To reduce the pence (1415) to shillings, we divide them by 12, and obtain 117 *shillings*, and have 11 *pence* over. These (11) pence, we place a little to the right, and bring down the 3 farthings that before remained still further to the right, as seen by the operation. In the same manner we proceed with the shillings, to bring them to pounds, we divide them by 20, and dispose of the remainder, together with all the preceding remainders, as directed above. Hence we have this

RULE.

97. Divide the *given denomination* by as many as it takes of the *same* to make ONE of the NEXT HIGHER; set the remainder, if any, to the right, and proceed as before.

From the illustrations, &c. now given, it evidently appears that,

98. REDUCTION—*Descending* and *Ascending* mutually prove each other.

TABLE 1.

ENGLISH MONEY.

4 farthings, (*far.*)	make	1 penny, (*d.*)
12 pence	"	1 shilling, (*s.*)
20 shillings	"	1 pound, (£)

LECTURE I.*

DESCENDING.

How do you reduce pounds to shillings? *Ans.* Multiply them by 20. Why? *Ans.* Because every pound is 20 shillings. How do you reduce shillings to pence? *Ans.* Multiply them by 12. Why? *Ans.* Because every shilling is 12 pence. How do you reduce pence to farthings? *Ans.* Multiply them by 4. Why? *Ans.* Because every penny is 4 farthings.

How do you reduce POUNDS, (*shillings, pence,* and *farthings*, if any,) to FARTHINGS?

Ans. Multiply the pounds by 20, to bring them to shillings, and add in the shillings, (if any,)—Multiply the shillings by 12, to bring them to pence, and add in the pence, (if any,)—Multiply the pence by 4, to bring them to farthings, and add in the farthings, (if any.)

EXAMPLES.

1. Reduce £15 to shillings. *Ans.* 300
2. Reduce 19*s.* to pence. 228
3. Reduce 11*d.* to farthings. 44
4. Reduce £15 19*s.* 11*d.* to farthings. *Ans.* 15356.

ASCENDING.

How do you reduce farthings to pence? *Ans.* Divide them by 4. Why? Because 4 farthings make 1 penny. What will the remainder be? *Ans.* Farthings. Why? Because the dividend is farthings. How do you reduce pence to shillings? *Ans.* Divide them by 12. Why? *Ans.* Because 12 pence make 1 shilling. What will the remainder be? *Ans.* Pence. Why? How do you reduce shillings to pounds? *Ans.* Divide them by 20. Why? *Ans.* Because 20 shillings make 1 pound. What will the remainder be? *Ans.* shillings. Why?

How do you reduce FARTHINGS to POUNDS, (*shillings, pence,* and *farthings*, if any?)

Ans. Divide the farthings by 4, to bring them to pence, and the remainder, (if any,) will be farthings—Divide the pence by twelve to bring them to SHILLINGS, and the remainder, (if any,) will be pence—Divide the shillings by 20, to bring them to POUNDS, and the remainder, (if any,) will be shillings.

EXAMPLES.

1. Reduce 44 *far.* to pence. *Ans.* 11
2. Reduce 228*d.* to shillings. 19
3. Reduce 300*s.* to pounds. 15
4. Reduce 15356 *far.* to pounds, &c. *Ans.* £15 19*s.* 11*d.*

TABLE 2.

TROY WEIGHT.†

24 grains, (*gr.* or *grs.*) make 1 pennyweight, (*pwt.*)
20 pennyweights " 1 ounce, (*oz.*)
12 ounces " 1 pound, (*lb.* or *lbs.*)

LECTURE II.‡

DESCENDING.

How do you reduce pounds to ounces? Why?—Ounces to pennyweights? Why?—pennyweights to grains? Why? How do you reduce POUNDS, (*ounces, pennyweights, and grains*, if any,) to GRAINS?

ASCENDING.

How do you reduce grains to pennyweights? Why?—pennyweights to ounces? Why?—ounces to pounds? Why? How do you reduce GRAINS to POUNDS, (*ounces, pennyweights,* and *grains*, if any?)

* These lectures will be found a great help in facilitating the study of reduction. To use them, let a class of scholars be seated with slates, or brought up before the *black board*, and let the teacher put the first question in the lecture to the first scholar, if he answers right, let him work out, on the slate or *black board*, the first of the examples that follow. But if he answers wrong, put the question to the next scholar. So proceed through the class. The *first examples* are, for this reason, very simple and easy, and exactly suited to the lectures.

† By this weight are weighed gold, silver, jewels, electaries and liquors. The fineness of gold is tried by fire, and is reckoned in CARATS; by which is meant the 24th part of any quantity; if it lose nothing in the trial, it is said to be 24 carats fine; if it lose 2 carats, it is then 22 carats fine, which is the standard for gold. Silver, which abides the fire without loss, is said to be 12 ounces fine. The standard for silver coin is 11oz. 2pwt. of fine silver, and 18pwt. of copper melted together.

‡ In this and the following lectures, the learner is expected to obtain the answers on the TABLES.

EXAMPLES.

1. Reduce 7*lbs.* to ounces. *Ans.* 84
2. Reduce 11*oz.* to *pwt.* 220
3. Reduce 19*pwt.* to grains. 456
4. Reduce 7*lbs.*11*oz.*19*pwt.* to grains. *Ans.* 46056.

EXAMPLES.

1. Reduce 456*grs.* to *pwt.* *Ans.* 19
2. Reduce 220*pwt.* to ounces. 11
3. Reduce 84*oz.* to pounds. 7
4. Reduce 46056*grs.* to pounds, &c. *Ans.* 7*lbs.* 11*oz.* 19*pwt.*

TABLE 3.

APOTHECARIES' WEIGHT.*

20 grains, (*gr.* or *grs.*)	make	1 scruple, (℈)
3 scruples	"	1 dram, (ʒ)
8 drams	"	1 ounce, (℥)
12 ounces	"	1 pound, (℔)

LECTURE III.

DESCENDING.

How do you reduce pounds to ounces?—ounces to drams?—drams to scruples?—scruples to grains?—POUNDS, (*ounces, drams, scruples,* and *grains,* if any,) to GRAINS?

EXAMPLES.

1. Reduce 25℔ to ounces. *Ans.* 300
2. Reduce 11 ℥ to drams. 88
3. Reduce 7 ʒ to scruples. 21
4. Reduce 2 ℈ to grains. 40
5. Reduce 25℔ 11 ℥ 7 ʒ 2 ℈ to grains. *Ans.* 149740.

ASCENDING.

How do you reduce grains to scruples?—scruples to drams?—drams to ounces?—ounces to pounds?—GRAINS to POUNDS, (*ounces, drams, scruples,* and *grains,* if any?)

EXAMPLES.

1. Reduce 40*grs.* to scruples. *Ans.* 2
2. Reduce 21 ℈ to drams. 7
3. Reduce 88 ʒ to ounces. 11
4. Reduce 300 ℥ to pounds. 25
5. Reduce 149740*grs.* to pounds, &c. *Ans.* 25℔ 11 ℥. 7 ʒ 2 ℈.

TABLE 4.

AVOIRDUPOIS WEIGHT.†

16 drams, (*dr.* or *drs.*)	make	1 ounce, (*oz.*)
16 ounces	"	1 pound, (*lb.* or *lbs.*)
28 pounds	"	1 quarter, (*qr.* or *qrs.*)
4 quarters	"	1 hundred weight, (*cwt.*)
20 hundred weight	"	1 ton, (*T.*)

LECTURE IV.‡

DESCENDING.

How do you reduce tons to hundred weight?—hundred weight to quarters?—quarters to pounds?—pounds to ounces?—ounces to drams?—TONS, (*hundred weight, quarters, pounds, ounces,* and *drams,* if any,) to DRAMS?

ASCENDING.

How do you reduce drams to ounces?—ounces to pounds?—pounds to quarters?—quarters to hundred weight?—hundred weight to tons?—DRAMS to TONS, (*hundred weight, quarters, pounds, ounces,* and *drams,* if any?)

* This weight is used by apothecaries in *compounding* their medicines. The pound and ounce this weight, and the pound and ounce Troy, are the same.

† By this weight are weighed all coarse and drossy goods; also all metals, except gold and silver. The method, however, of estimating 28 pounds to a quarter, is growing out of use; the true quarter, 25 pounds, being substituted in its stead. A table may be constructed containing the true hundred and quarter, by simply erasing 28 in the above, and inserting 25. 175*oz.* Troy = 192*oz.* Avoir., and 175*lbs.* Troy = 144*lbs.* Avoir.

‡ Here and subsequently the WHYS are omitted; but the teacher may easily supply them.

EXAMPLES.

1. Reduce 93 *T.* to *cwt.* *Ans.* 1860
2. Reduce 19*cwt.* to *qrs.* 76
3. Reduce 3*qrs.* to pounds. 84
4. Reduce 27*lbs.* to ounces. 432
5. Reduce 15*oz.* to drams. 240
6. Reduce 93 *T.* 19*cwt.* 3*qrs.* 27*lbs.* 15*oz.* to drams. *Ans.* 53903344.

EXAMPLES.

1. Reduce 1860*cwt.* to tons. *Ans.* 93
2. Reduce 76*qrs.* to *cwt.* 19
3. Reduce 84*lbs.* to quarters. 3
4. Reduce 432*oz.* to pounds. 27
5. Reduce 240*drs.* to ounces. 15
6. Reduce 53903344*drs.* to tons, &c. *Ans.* 93 *T.* 19*cwt.* 3*qrs.* 27*lbs.* 15*oz.*

TABLE 5.

CLOTH MEASURE.

4 nails, (*na.*)	make	1 quarter, (*qr.* or *qrs.*)
4 quarters	"	1 yard, (*yd.* or *yds.*)
5 quarters	"	1 Ell English, (*E. E.*)
3 quarters	"	1 Ell Flemish, (*E. Fl.*)
6 quarters	"	1 Ell French, (*E. Fr.*)

LECTURE V.

DESCENDING.

How do you reduce yards to quarters?—quarters to nails?—YARDS (*quarters* and *nails*, if any,) to NAILS?—Ells English to quarters?—Ells Flemish to quarters?—Ells French to quarters?—Ells English to nails?

ASCENDING.

How do you reduce nails to quarters?—quarters to yards?—NAILS to YARDS, (*quarters* and *nails*, if any?)—quarters to Ells English?—quarters to Ells Flemish?—quarters to Ells French?—nails to Ells English.

EXAMPLES.

1. Reduce 17 *yards* to quarters. *Ans.* 68
2. Reduce 3 *qrs.* to nails. 12
3. Reduce 17 *yds.* 3 *qrs.* to nails. 284
4. Reduce 21 *E. E.* to qrs. 105
5. Reduce 13 *E. Fl.* to qrs. 39
6. Reduce 19 *E. Fr.* to qrs. 114
7. Reduce 37 *E. E.* to nails. 740

EXAMPLES.

1. Reduce 12 *na.* to quarters. 3
2. Reduce 68 *qrs.* to yards. 17
3. Reduce 284 *nails* to yards, &c. *Ans.* 17 *yds.* 3 *qrs.*
4. Reduce 105 *qrs.* to E. E. 21
5. Reduce 39 *qrs.* to E. Fl. 13
6. Reduce 114 *qrs.* to E. Fr. 19
7. Reduce 740 *na.* to E. E. 37

TABLE 6.

LONG MEASURE.*

3 barley corns, (*b. c.*)	make	1 inch, (*in.*)
12 inches	"	1 foot, (*ft.*)
3 feet	"	1 yard, (*yd.* or *yds.*)
5½ yards	"	1 rod, perch or pole, (*rd.*, *rds.*, *p.*)
40 rods	"	1 furlong, (*fur.*)
8 furlongs	"	1 mile, (*m.*)
69½ miles	"	1 degree, (*deg.*)

LECTURE VI.

DESCENDING.

How do you reduce degrees to miles?—miles to furlongs?—furlongs to rods?—

ASCENDING.

How do you reduce barley corns to inches?—inches to feet?—feet to yards?

* Used in measuring any thing in which *length* only is considered. A league is 3 miles. This measure is chiefly used in measuring distances on water. A degree is 69½ statute miles, or 60 geographic miles. The circumference of the earth is 360 degrees.

rods to yards ?—yards to feet ?—feet to inches ?—inches to barley corns—DEGREES, (*miles, furlongs, rods, yards, feet, inches,* and *barley corns*, if any,) to BARLEY CORNS ?

EXAMPLES.

1. Reduce 15 *deg.* to miles. *Ans.* 1042½.
2. Reduce 43 *miles* to furlongs. 344
3. Reduce 7 *fur.* to rods. 280
4. Reduce 35 *rods* to yards. 192½
5. Reduce 5 *yds.* to feet. 15
6. Reduce 2 *ft.* to inches. 24
7. Reduce 11 *in.* to *b. c.* 33
8. Reduce 15*deg.* 43*m.* 7*fur.* 35*rds.* 5*yds.* 2*ft.* 11*in.* to barley corns.
9. Reduce 17*deg.* 25*m.* 3*fur.* 15*rds.* 2*yds.* 1*ft.* 10*in.* 2*b. c.* to barley corns. *Ans.* 229411994.

—yards to rods ?—rods to furlongs ?—furlongs to miles ?—miles to degrees ?—BARLEY CORNS TO DEGREES, (*miles, furlongs, rods, yards, feet, inches,* and *barley corns*, if any ?)

EXAMPLES.

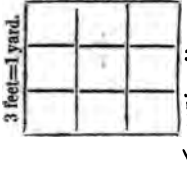
1. Reduce 33*b. c.* to inches. *Ans.* 11
2. Reduce 24*in.* to feet. 2
3. Reduce 15*ft.* to yards. 5
4. Reduce 192½*yds.* to rods. 35
5. Reduce 280*rods* to furlongs. 7
6. Reduce 344*fur.* to miles. 43
7. Reduce 1042½*m.* to degrees. *Ans.* 15

TABLE 7.

LAND, OR SQUARE MEASURE.*

144 square inches, (*in.*) make 1 square foot, (*ft.*)
9 " feet " 1 " yard, (*yd.* or *yds.*)
30¼ " yards " 1 " rod, (*rd.* or *rds.*)
40 " rods " 1 " rood, (*R.*)
4 " roods " 1 " acre, (*A.*)

ILLUS. Under ex. 35, page 35, we have defined a *square* to be a figure having ALL its sides *equal and parallel*; and from the illustration there given it is evident that we obtain the number of *square feet* in a *square yard* by multiplying the number of feet in a yard (table 6) INTO ITSELF. Because, since there are *three* rows, 3 feet long and 1 foot wide, there must be in each row, 3 *square feet*, and therefore in THREE rows, 3×3=9 *square feet*. Hence it is evident, that 144 square inches make 1 square foot, because 1 square foot is 12 inches one way, and 12 the other, and 12×12=144.



LECTURE VII.

DESCENDING.

How do you reduce acres to roods ?—roods to rods ?—rods to yards ?—yards to feet ?—feet to inches ?—ACRES, (*roods, rods, yards, feet,* and inches, if any,) to INCHES ?

EXAMPLES.

1. Reduce 7 *A.* to roods. *Ans.* 28
2. Reduce 3 *R.* to rods. 120
3. Reduce 39 *rds.* to yards.† 1179¾
4. Reduce 27 *yds.* to feet. 243
5. Reduce 5 feet to inches. 720
6. Reduce 7*A.* 3*R.* 39*rd.* 27*yds.* 5*ft.* to inches. *Ans.* 50177628.

ASCENDING.

How do you reduce inches to feet ?—feet to yards ?—yards to rods ?—rods to roods ?—roods to acres ?—INCHES TO ACRES, (*roods, rods, yards, feet,* and *inches*, if any ?)

EXAMPLES.

1. Reduce 720*in.* to feet. *Ans.* 5
2. Reduce 243*ft.* to yards. 27
3. Reduce 1179¾*yds.* to rods.‡ 39
4. Reduce 120*rds.* to roods. 3
5. Reduce 28*R.* to acres. 7
6. Reduce 50177628*in.* to acres, &c. *Ans.* 7*A.* 3*R.* 39*rds.* 27½*yds.* 2*ft.* 108*in.*

* Used in finding the contents of land, or of any thing else, in which *length* and *breadth* are considered, without regard to thickness.

† See Art. 81. *Illus.* 2. ‡ See Art. 69.

TABLE 8.

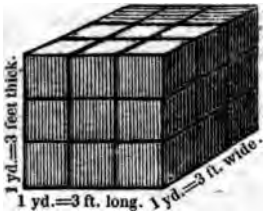
SOLID, OR CUBIC MEASURE.*

1728 solid or cubic inches, (*in.*) make 1 solid or cubic foot, (*ft.*)
27 " " feet " 1 " " yard, (*yd.* or *yds.*)
128 " " feet " 1 cord, (*C.*)

NOTE. 40 feet of round timber, or 50 feet of hewn timber, make 1 ton or load.

DEFINITION 1. A SOLID is any thing that has *length*, *breadth*, and *thickness*.

2. A CUBE is a solid having its *three dimensions* ALL EQUAL.



ILLUS. We have seen, (Table 7,) that a square yard contains (3 × 3 =) 9 square feet. A cubic yard is 3 feet long, 3 feet wide, and 3 feet thick. Were it 3 feet long, 3 feet wide, and ONE foot thick, it would contain 9 cubic feet; but as it is 3 feet thick it does contain 3 × 9 = 27 cubic feet. That is, we obtain the solid contents by multiplying the *three dimensions* together, thus, 3×3×3 = 27 solid or cubic feet. *Ans.*

Hence, also, 1728 cubic inches make 1 cubic foot, because 1 cubic foot is 12 *inches* long, 12 in. wide, and 12 in. thick; and, 12×12×12=1728.

LECTURE VIII.

DESCENDING.

How do you reduce cords to feet?—feet to inches?—CORDS, (*feet* and *inches*, if any,) to *inches*?—YARDS, (*feet* and *inches*, if any,) to INCHES?

EXAMPLES.

1. Reduce 5*C.* to feet. *Ans.* 640
2. Reduce 57*ft.* to inches. 98496
3. Reduce 5*C.* 57*ft.* to inches. *Ans.* 1204416.

ASCENDING.

How do you reduce inches to feet?—feet to cords?—INCHES to CORDS?—(*feet* and *inches*, if any?)—feet to yards?—INCHES to YARDS, (*feet* and *inches* if any?)

EXAMPLES.

1. Reduce 98496 *in.* to feet. *Ans.* 57
2. Reduce 640 *ft.* to cords. 5
3. Reduce 1204416 *in.* to cords, &c. *Ans.* 5*C.* 57*ft.*

TABLE 9.

WINE MEASURE.†

4 gills, (*g.*) make 1 pint, (*pt.* or *pts.*)
2 pints " 1 quart, (*qt.* or *qts.*)
4 quarts " 1 gallon, (*gal.*)
31½ gallons " 1 barrel, (*bbl.*)
2 barrels " 1 hogshead, (*hhd.*)
2 hogsheads " 1 pipe, (*P.*)
2 pipes " 1 tun, (*T.*)

NOTE. The wine gallon contains 231 cubic inches.

* Used in finding the contents of things that have *length*, *breadth*, and *thickness*.

† Used in measuring the various kinds of liquors; such as beer, cider, wine, &c. &c. A hogshead contains 63 gallons; for it contains 2 barrels of 31½ gallons each, and 31½×2=63.

Beer Measure, being seldom used in this country, has been omitted in the text. But as some may have use for this measure, we give it the following insertion.

ALE, OR BEER MEASURE.

2 pints, (*pt.* or *pts.*) make 1 quart, (*qt.* or *qts.*)
4 quarts " 1 gallon, (*gal.*)
36 gallons " 1 barrel, (*bbl.*)
54 gallons " 1 hogshead, (*hhd.*)
3 barrels " 1 butt, (*B.*)

LECTURE IX.

DESCENDING.

How do you reduce tuns to pipes?—pipes to hogsheads?—hogsheads to barrels?—barrels to gallons?—gallons to quarts?—quarts to pints?—pints to gills?—TUNS, (*pipes, hogsheads, barrels, gallons, quarts, pints* and *gills*, if any,) to GILLS?

EXAMPLES.

1. Reduce 3 tuns to pipes. *Ans.* 6
2. Reduce 1*P.* to hogsheads. 2
3. Reduce 1*hhd.* to barrels. 2
4. Reduce 1*bbl.* to gallons. 31½
5. Reduce 17*gal.* to quarts. 68
6. Reduce 3*qts.* to pints. 6
7. Reduce 1*pt.* to gills. 4
8. Reduce 3*T.* 1*P.* 1*hhd.* 1*bbl.* 17*gal.* 3*qts.* 1*pt.* to gills.
   Ans. 31820.

ASCENDING.

How do you reduce gills to pints?—pints to quarts?—quarts to gallons?—gallons to barrels?—barrels to hogsheads?—hogsheads to pipes?—pipes to tuns?—GILLS to TUNS, (*pipes, hogsheads, barrels, gallons, quarts, pints* and *gills*, if any?)

EXAMPLES.

1. Reduce 4 gills to *pt.* *Ans.* 1
2. Reduce 6*pts.* to quarts. 3
3. Reduce 68*qts.* to gallons. 17
4. Reduce 31½*gal.* to *bbl.* 1
5. Reduce 2*bbl.* to *hhd.* 1
6. Reduce 2*hhd.* to *P.* 1
7. Reduce 6*P.* to tuns. 3
8. Reduce 31820 gills to tuns, &c.
   Ans. 3*T.* 1*P.* 1*hhd.* 1*bbl.* 17½*gal.* 1*qt.* 1*pt.*

TABLE 10.

DRY MEASURE.

2 pints, (*pt.* or *pts.*)	make	1 quart, (*qt.* or *qts.*)
8 quarts	"	1 peck, (*pk.* or *pks.*)
4 pecks	"	1 bushel, (*B.*)

NOTE.—The bushel measure contains 2150⅖ cubic inches = a vessel 18½*in.* diameter, and 8*in.* deep.

LECTURE X.

DESCENDING.

How do you reduce bushels to pecks?—pecks to quarts?—quarts to pints?—BUSHELS, (*pecks, quarts* and *pints*, if any,) to PINTS?

EXAMPLES.

1. Reduce 15*B.* to pecks. *Ans.* 60
2. Reduce 3*pks.* to quarts. 24
3. Reduce 7*qts.* to pints. 14
4. Reduce 15*B.* 3*pks.* 7*qts.* to pints.
   Ans. 1022.

ASCENDING.

How do you reduce pints to quarts?—quarts to pecks?—pecks to bushels?—PINTS to BUSHELS, (*pecks, quarts* and *pints*, if any?)

EXAMPLES.

1. Reduce 14*pts.* to quarts. *Ans.* 7
2. Reduce 24*qts.* to pecks. 3
3. Reduce 60*pks.* to bushels. 15
4. Reduce 1022*pts.* to bushels, &c.
   Ans. 15*B.* 3*pks.* 7*qts.*

TABLE 11.

TIME.

60 seconds, (*sec.*)	make	1 minute, (*m.*)
60 minutes	"	1 hour, (*h.*)
24 hours	"	1 day, (*d.*)
7 days	"	1 week, (*w.*)
4 weeks	"	1 lunar month, (*mo.*)
13 lunar months*	"	1 year, (*Y.*)

* Or, more exactly, 13 lunar months, 1 day, and 6 hours.

LECTURE XI.

DESCENDING.

How do you reduce years to months?—months to weeks?—weeks to days?—days to hours?—hours to minutes?—minutes to seconds?—YEARS, (*months, weeks, days, hours, minutes* and *seconds*, if any,) to SECONDS?

EXAMPLES.

1. Reduce 14 *Y.* to months. *Ans.* 168
2. Reduce 11*mo.* to weeks. 44
3. Reduce 3*w.* to days. 21
4. Reduce 5*d.* to hours. 120
5. Reduce 15*h.* to minutes. 900
6. Reduce 27*m.* to seconds. 1620
7. Reduce 14 *Y.* 11*mo.* 3*w.* 5*d.* 15*h.* 27*m.* to seconds.

Ans. 435338820.

ASCENDING.

How do you reduce seconds to minutes?—minutes to hours?—hours to days?—days to weeks?—weeks to months?—months to years?—SECONDS to YEARS, (*months, weeks, days, hours, minutes* and *seconds*, if any?)

EXAMPLES.

1. Reduce 1620*sec.* to *m.* *Ans.* 27
2. Reduce 900*m.* to hours. 15
3. Reduce 120*h.* to days. 5
4. Reduce 21*d.* to weeks. 3
5. Reduce 44*w.* to months. 11
6. Reduce 168*mo.* to years. 14
7. Reduce 435338820*sec.* to years, &c. *Ans.* 14 *Y.* 11*mo.* 3*w.* 5*d.* 15*h.* 27*m.*

99. TIME is also reckoned by the calendar—dividing the year into 12 *calendar months*, containing 365 days, as follows:

mo.	name.	d's.	mo.	name.	d's.	mo.	name.	d's.	mo.	name.	d's.
1st	Jan.	31	2nd	Feb.	28	3nd	March	31	4th	April	30
5th	May	31	6th	June	30	7th	July	31	8th	Aug.	31
9th	Sept.	30	10th	Oct.	31	11th	Nov.	30	12th	Dec.	31

The exact year is estimated at 365*d.* 5*h.* 48*m.* 48*sec.*; but 365*d.* 6*h.* will be sufficiently exact for common purposes. The 6*h.* are reserved till they amount to a whole day, which is every 4th year, and then this day is added to the month of February, which then has 29 days. Hence every fourth year contains 366 days, and is called *leap-year*. Therefore, to know when it is leap-year,

RULE. *Divide the year by* 4, *and if nothing remain, it is leap-year.*

EX. For the year 1832. 4)1832(458 & 0 remains, therefore it is leap-year.

The number of days in each month may be remembered by the following lines:

Thirty days hath September,
April, June, and November—
All the rest have thirty-one,
Excepting February 'lone,
Which has but twenty-eight, in fine,
Till leap year gives it twenty-nine.

100. The first seven letters of the alphabet, A, B, C, D, E, F, G, are used to mark the several days of the week; the first letter, A, standing for the first day of Jan., B for the second, and so on. Since, therefore, one of these letters must necessarily stand for Sunday, it is called the DOMINICAL, or Sunday Letter.

When Jan. begins on Sunday, the *Dominical Letter* is A; and since the next year must begin on Monday, Sunday will be the 7th, and G the dominical letter; and as the third year begins on Tues., Sunday will be the 6th, and F the dominical letter. Hence the dominical letters succeed each other in the retrograde order, G, F, E, D, C, B, A: and, were it not for leap-year, the same day of the week would, years, return to be the same day of the month. But this interruption makes it 4 7 = 28 years.

1. Wherefore, *to find the* DOMINICAL LETTER, we have this

RULE. To the given year add its FOURTH PART ;* divide the sum by 7, and the *remainder* will be the INDEX of the letter.

EX. Required the DOMINICAL LETTER for 1831?

1	2	3	4	5	6	7
Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
INDICES 0	1	2	3	4	5	6
A	G	F	E	D	C	B

Year 1831
Its 4th part 457
Days in the week, 7)2288
Rem. 326 — 6 index answering to B, the D. L. required.

NOTE. Every *leap-year* has two dominical letters; the one found by the rule serves for March and the subsequent months, and the next in retrograde order, for the rest of the year.

102. Having found the Dominical Letter, *the day of the* WEEK *on which any* GIVEN DAY *of the* MONTH *will fall*, may be found by the following couplet:

1 Jan.	2 Feb.	3 March.	4 April.	5 May.	6 June.
AT	DOVER	DWELLS	GEORGE	BROWN,	ESQUIRE,
GOOD	CALEB	FRINK	AND	DAVID	FRIAR.
7 July.	8 Aug.	9 Sept.	10 Oct.	11 Nov.	12 Dec.

The first letter of each word marks the day of the week on which each corresponding month comes in; whence any other day may be easily found.

EXAMPLE. What day of the week will be the 4th of July, 1831?

The dominical letter for the given year is B. *Good* answers to July; therefore, reckoning, (B, C, D, E, F,) G, we find that July comes in on (*Sun.*, *Mon.*, *Tues.*, *Wed.*, *Thurs.*,) FRIDAY; hence the 4th day falls on MONDAY.

The result, however, is more readily obtained by counting the fingers. Thus, count the fingers up to the given month, and *mark that finger*: then repeat the words of the above couplet to the *same finger*, and note the INITIAL; then begin with the *Dominical Letter*, and say the letters over to *that same initial*; finally, tell the days of the week over to THE SAME FINGER, and it will give the day required.

TABLE 12.

CIRCULAR MOTION.

60 seconds (″)	make	1 minute, (′)
60 minutes	"	1 degree, (°)
30 degrees	"	1 sign, (S.)
12 signs	"	1 circle, (C.) or complete revolution of the Zodiac.

NOTE. All circles are supposed to be divided into 360 equal parts, called *degrees*.

LECTURE XII.

DESCENDING.

How do you reduce circles to signs?—signs to degrees?—degrees to minutes?—minutes to seconds?—CIRCLES, (*signs*, *degrees*, *minutes*, and *seconds*, if any,) to SECONDS?

ASCENDING.

How do you reduce seconds to minutes?—minutes to degrees?—degrees to signs?—signs to circles?—SECONDS to CIRCLES, (*signs*, *degrees*, *minutes*, and *seconds*, if any?)

* Omitting fractions. If 0 remain the dominical letter is A.

EXAMPLES.		EXAMPLES.	
1. Reduce 3C. to signs.	*Ans.* 36	1. Reduce 900″ to minutes.	*Ans.* 15
2. Reduce 7S. to degrees.	21	2. Reduce 1200′ to degrees.	20
3. Reduce 20° to minutes.	120	3. Reduce 210° to signs.	7
4. Reduce 15′ to seconds.	900	4. Reduce 36S. to circles.	3
5. Reduce 3C. 7S. 20° 15′ to seconds.		5. Reduce 4716900″ to degrees, &c.	
Ans. 4716900.		*Ans.* 3C. 7S. 20° 15′	

Measures, &c. not included in the foregoing Tables.

The following is used by surveyors in measuring land:

7 $\frac{92}{100}$ inches, (*in.*) make 1 link, (*l.*)
25 links " 1 rod, (*rd.* or *rds.*)
4 rods or 66 feet " 1 chain, (*ch.*)
10 square chains " 1 acre, (*A.*)

6 points make 1 line, } used in measuring the length of the
12 lines " 1 inch, } rods of clock pendulums.
4 inches " 1 hand, } used in measuring the height of horses.
6 feet " 1 fathom, used in measuring depths at sea.

12 particular things make 1 dozen,
12 dozen " 1 gross,
12 gross " 1 great gross.

Also,

20 particular things make 1 score.
112 pounds make one quintal of fish.

24 sheets of paper make 1 quire,
20 quire " 1 ream.

FEDERAL MONEY.

10 mills, (*m.*) make 1 cent, (*ct.* or *cts.*)
10 cents " 1 dime, (*d.*)
10 dimes " 1 dollar, (*dol.* or $.)
10 dollars " 1 eagle, (*E.*)

But this money is commonly divided into *dollars*, *cents*, and *mills* only, reckoning 100 cents to a dollar, and 10 mills to a cent.

☞ As *Federal Money* is purely decimal, it will be incorporated with Decimal Fractions. It is a great mistake to suppose that this money "is so nearly allied to whole numbers," as to admit of being incorporated with them; because, if this argument will hold true with regard to *federal money*, it must also hold true with regard to all other decimals, for all other decimals bear the same relation.

THEORETICAL QUESTIONS.

What does the word *denomination* signify? What are compound numbers? What is the difference between a simple and a compound number? What is reduction? How is it divided? What is reduction descending? How is it performed? What is reduction ascending? How is reduction ascending performed? What is the general rule for reduction descending? What, for reduction ascending?

How is time commonly reckoned? What is the exact year? How can we tell when it is leap-year? What are the first 7 letters of the alphabet used for? What is the rule for finding the dominical letter? Having found the dominical letter, how can we find the day of the week on which any given day of the month will

On what day of the week, in 1832, will the 21st of Feb. fall? —— 17th
ov.? —— 27th of August?

EXAMPLES FOR PRACTICE.

1. In £17 12*s.* 5*d.* how many farthings? *Ans.* 16916.
2. In 7085 *pwt.* how many pounds, &c.? *Ans.* 29*lbs.* 6*oz.* 5 *pwts.*
3. In 2*lbs.* 8*oz.* 8*pwt.* of silver, how many table spoons, weighing 22*pwt.* each, and tea spoons, each 5*pwt.* may be made, and an equal number of each sort? *Ans.* 24.
4. In 9*cwt.* 3*qrs.* 13*lbs.* of indigo, how many ounces? *Ans.* 17680.
5. In 4lb. 5 ʒ. how many drams? *Ans.* 424.
6. In 500 *nails*, how many Ells English? *Ans.* 25.
7. The highest mountain in the known world is the Himmaleh in Asia, which is 5*m.* 0*fur.* 3*rds.* 3½*yds.* 2*ft.*, how many feet is that? *Ans.* 26462.
8. The next highest is Chimborazo, in South America, which is 21450 feet, how many miles is that? *Ans.* 4*m.* 0*fur.* 20*rds.*
9. In 2560 square rods, how many acres? *Ans.* 16.
10. In 2233 feet, how many cords, &c.? *Ans.* 17*C.* 57*ft.*
11. How many *two quart* bottles, *quart* bottles, and *pint* bottles, of each an equal number, may be filled from a pipe of wine? *Ans.* 144.
12. In 242 pints, how many bushels, &c.? *Ans.* 3*bu.* 3*pks.* 1*qt.*
13. The latitude of Boston is 42° 23′ 2″ north; how many seconds is that? *Ans.* 152582.
14. The latitude of Rio Janeiro amounts to 82739″ south, how many degrees, &c.? *Ans.* 22° 58′ 59″
15. In a silver tankard, weighing 3*lbs.* 11*oz.* 19*pwt.* 23*gr.*, how many grains? *Ans.* 23039.
16. The distance from New-York to Albany is 141 miles, how many inches is that? *Ans.* 8933760.
17. In a hogshead of tobacco, weighing 5*cwt.* 3*qrs.* 26*lbs.*, how many pounds? *Ans.* 670.
18. A boy found a gold medal, weighing 3*oz.* 17*pwt.* 21*grs.*, and finding no owner, he disposed of it at 1 penny per grain. What did he get for it? *Ans.* £7 15*s.* 9*d.*
19. In 256 boxes of raisins, each weighing 19*lbs.*, how many hundred weight, &c? *Ans.* 43*cwt.* 1*qr.* 20*lbs.*
20. In 1lb. 2ʒ. 2ʒ. 1℈., how many scruples? *Ans.* 343.
21. The distance from the earth to the sun is 95,000,000 of miles; how long would a cannon ball be in passing that distance at the rate of 8 miles per minute? *Ans.* 22*y.* 216*d.* 12*h.* 40*m.*
22. How many inches will reach round the globe, it being 360 degrees? *Ans.* 1585267200.
23. If the British national debt be 13 hundred millions of pounds sterling, how long would it take to count it, at the rate of £50 a minute, reckoning without intermission 12 hours each day, and 365 days to the year? *Ans.* 98*y.* 341*d.* 1*h.* 20*m.*

ADDITION OF COMPOUND NUMBERS,

COMMONLY CALLED

COMPOUND ADDITION.

103. COMPOUND ADDITION is finding the SUM of two or more COMPOUND NUMBERS.

	£.	s.	d.
Sold 1 pr. of gloves,	0	5	9
. . . 1 hndkf.	0	7	6
What is the sum? *Ans.*	£0	13	3

6*d.* and 9*d.* is 15*d.*; 15*d.* = 1*s.* 3*d.* (Art. 96.) Set down the 3*d.* under the column of *pence*, and carry the 1*s.* to the next column, saying 1 to 7 is 8, and 5 is 13*s.* This set down, gives for the answer, 13*s.* 3*d.* The

process. It will be seen, common in adding up each column, dividing the sum by as many as of the same to make one of the next higher, setting down the remainder, and carrying the quotient.

104. Hence, for the addition of compound numbers, we have this

RULE I. Place the numbers so that the same denominations may stand directly under each other.

II. Add up the *numbers of each denomination*, reduce their sum to the next higher, set down the remainder, and carry the quotient.

PROOF. The same as in simple addition.

ENGLISH MONEY.

£	s.	d.	qrs.
249	14	5	1
157	19	4	3
176	15	1	2
542	12	6	1

2. A merchant began trade with £1755 13*s.* 4*d* in goods, £459 12*s.* 3*d.* in good debts,—and he cleared the first year £249 19*s.* 10*d.*; what was he worth at the year's end? *Ans.* £2465 5*s.* 5*d.*

TROY WEIGHT.

lbs.	oz.	pwt.	grs.
7	11	19	20
6	5	17	15
5	11	13	21
9	7	14	23

4. If an ingot of gold weigh 7*lbs.* 9*oz.* 11*pwt.* 19*grs.*, a gold medal 4*oz.* 17*pwt.* 21*grs.*, and another lump 5*lbs.* 6*oz.* 17*pwt.* 6*grs.*, what is the whole weight? *Ans.* 13*lbs.* 11*oz.* 6*pwt.* 22*grs.*

APOTHECARIES' WEIGHT.

lb.	ʒ	℈	ℌ	grs.
9	11	7	2	19
3	7	6	0	23
7	9	5	1	2
5	10	4	2	18

6. An apothecary would make a compound of 3 ingredients; of the first he would put 11 ʒ 7 ℈ 2 ℌ 19*grs.*, of the second 2*lb.*, and of the third 6 ʒ 2 ℈ 23*grs.*; how much in the whole? *Ans.* 3*lb.* 0 ʒ 6 ℈ 0 ℌ 18*grs.*

AVOIRDUPOIS WEIGHT.

T.	cwt.	qrs.	lbs.	oz.	drs.
15	7	2	25	14	8
17	5	3	19	15	7
13	3	0	27	9	6
19	0	1	13	4	13

8. Bought tobacco 6*cwt* 2*qrs.* 27*lbs.*, raisins 23*cwt.* 1*qr.* 20*lbs.*, sugar 11*cwt.* 18*lbs.*; what is their amount? *Ans.* 41*cwt.* 1*qr.* 9*lbs.*

CLOTH MEASURE.

Yds.	qrs.	na.
175	1	3
191	3	1
144	0	2
365	2	3

10. Bot. 3 pieces of cloth, one containing 72*yds.* 3*qrs.* 2*na.*, another 87*yds.* 1*qr.* 3*na.*, and a third 54*yds.* 2*na.*; how much in all? *Ans.* 214*yds.* 1*qr.* 3*na.*

LONG MEASURE.

	deg.	m.	fur.	rds.	yds.	ft.	in.	&c.
11.	17	35	7	26	5	2	11	2
	16	59	5	32	4	1	9	1
	5	28	3	39	2	0	10	2
	13	37	4	18	3	2	8	2

LAND OR SQUARE MEASURE.

A.	R.	rds.
18	3	29
32	2	17
27	3	21
15	1	15

13. A man owns 87*A.* 2*R.* 27*rds.*; he buys a piece adjoining, 32*A.* 16*rds.*, another, 67*A.* 3*R.* 21*rds.*, and a third, 21*A.* 2*R.* 96*rds.*; how much did he then own? *Ans.* 219*A.* 1*R.* 20*rds.*

SOLID OR CUBIC MEASURE.

	C.	ft.	in.
14.	13	75	187
	17	36	317
	19	67	703
	25	32	980

15. Sold to one man 3 loads of wood; 1st load 57*ft.* 1142*in.*, 2d load 49*ft.* 1327*in.*, 3d load 62*ft.* 1284*in.*; how much in all?

Ans. 1*C.* 42*ft.* 297*in.*

WINE MEASURE.

	Bbl.	gal.	qts.
16.	17	15	3
	26	17	2
	19	28	1
	13	21	0

17. A man has 4 casks, containing as follows: 1st. 28*gal.* 3*qts.* 1*pt.*, 2d 27*gal.* 2*qts.*, 3d 29*gal.* 1*pt.*, and 4th 15*gal.* 3*qts.* 1*pt.*; how much will they all hold?

Ans. 3*bbl.* 6*gal.* 3*qts.* 1*pt.*

DRY MEASURE.

	B.	pks.	qts.
18.	16	3	7
	14	2	3
	17	3	6
	19	2	5

19. Bot. 3 parcels of grain, one containing 21*B.* 3*pks.* 1*qt.*, another 17*B.* 2*qts.*, a third 19*B.* 2*pks.* 7*qts.*; how much in all?

Ans. 58*B.* 2*pks.* 2*qts.*

TIME.

	Y.	mo.	w.	d.	h.	m.	sec.
20.	14	11	3	6	23	59	59
	13	10	2	5	22	58	58
	12	9	1	4	21	57	57
	11	8	0	3	20	56	56

THEORETICAL QUESTIONS. What is compound addition? In what does it consist? What is the first part of the rule? What the second? —— the PROOF?

SUBTRACTION OF COMPOUND NUMBERS,

COMMONLY CALLED

COMPOUND SUBTRACTION.

105. COMPOUND SUBTRACTION is finding the DIFFERENCE between two COMPOUND NUMBERS.

	£	s.	d.	
1. Henry has	0	15	9	How much has Henry more than Charles?
Charles "	0	10	6	
Difference	£0	5*s.*	3*d.*,	*Ans.*

Here it is plain that we find the difference, for 6d. from 9d. leaves 3d., and 10s. from 15s. leaves 5s. Therefore Henry evidently has 5s. 3d. the most.

	£	s.	d.	
2. Oliver has	0	17	6	How much has Oliver more than Cyrus?
Cyrus "	0	13	9	
Diff.	£0	3*s.*	9*d.*,	*Ans.*

Here we cannot take 9d. from 6d., but we can borrow one of Oliver's (17) shillings, and taking 9d. from this, the rem. is 3d.; 3 and 6 (the number of *pence* above) is 9; set down 9d. Now as we have borrowed 1 from the 17s., it is evident that we must either call 17 *one less*, or carry 1 to 13 (the number below,) and take the SUM from 17. The latter mode is most convenient.

106. Hence, TO SUBTRACT COMPOUND NUMBERS, we have this

RULE I. Place the LESSER number under the GREATER, and the SAME DENOMINATIONS under EACH OTHER.

II. Begin at the right hand, and take the number of each denomination in the lower line from the number above it; and set the remainder underneath.

III. If the number in the lower line EXCEED that above it, take it from AS MANY as it takes of that denomination to make one of the NEXT HIGHER, and add the remainder to the upper number; set the result underneath, CARRY ONE to the next lower number, and proceed as before.

PROOF. The same as in simple subtraction.

EXAMPLES.

1. In account with Wm. Trusty on my book, he stands Dr. to £112 8*s.* 3*d.*, Cr. to £87 15*s.* 9*d.*; what is the balance, and in whose favour?

2. Purchased an ingot of gold, weighing 5*lbs.* 4*oz.* 17*pwt.*; sold 2*lbs.* 11*oz.* 6*pwt.*; how much is there left? and what will it come to at 3*d.* per grain? *Ans. to the last,* £177 6*s.*

3. Bot. 13*cwt.* 15*lbs.* of sugar; sold 5*cwt.* 2*qrs.* 23*lbs.*; what will the balance come to, at 10*d.* a pound? *Ans.* £34 13*s.* 4*d.*

4. Bot. muslin, 217*yds.* 2*qrs.*; sold 158*yds.* 3*qrs.* 2*na.*; how much remains? *Ans.* 58*yds.* 2*qrs.* 2*na.*

5. A certain town is 21*m.* 3*fur.* 25*rds.* long, and 15*m.* 5*fur.* 17*rds* wide; how much longer is it than wide? *Ans.* 5*m.* 6*fur.* 8*rds.*

6. A man has a farm of 217*A.* 2*R.* 12*rds.*; he sells 39*A.* 3*R.* 33*rds.*; how much has he left? *Ans.* 177*A.* 2*R.* 19*rds.*

7. From a lot of wood containing 526*C.* 72*ft.* 1017*in.*, sold 87*C.* 98*ft.* 1125*in.*; how much remains? *Ans.* 438*C.* 22*ft.* 1620*in.*

8. Bot. a cask of wine containing 57*gal.* 3*qts.* 1*pt.*; out of which I sold 19*gal.* 2*qts.*; how much remains? *Ans.* 38*gal.* 1*qt.* 1*pt.*

9. Put up in my bin, 736*B.* 1*pk.* of wheat; and having sold 379*B.* 3*pks.*; how much remains? *Ans.* 356*B.* 2*pks.*

	deg.	m.	fur.	rds.	yds.	ft.	in.	b.c.	
10. From	38	41	3	29	2	1	7	1	} and prove the result.
Subtract	19	35	5	31	3	1	9	2	}

11. My note against Peter Pencil, given May 20th, 1825, is this day to be settled—April 21, 1828: For how long time must interest be reckoned?

Y.	mo.	d.
1828	4	21
1825	5	20
Ans. 2	11	1

We begin with the last date, and set down the year (1828), *mo.* of the month (4), and day of the month (21). Then with the first date, we set down the year (1825), *mo.* of the month (5), and day of the month (20) UNDERNEATH; and, subtracting, obtain 2*y.* 11*mo.* 1*d.*, the answer.

107. Hence, *to obtain the* DIFFERENCE OF TIME *between any two given dates,* we have this

RULE. Begin with the last date, and set down the YEAR, *number of the month,* and *day of the month;* under which set down the first date in the SAME WAY, and subtract as before directed.

NOTE. In casting interest, each month is reckoned 30 days.

12. What difference of time between Sept. 18, 1821, and June 16, 1828? *Ans.* 6*y.* 8*mo.* 28*d.*

13. Between Aug. 17, 1815, and June 1, 1829? *Ans.* 13*y.* 4*mo.* 15*d.*

14. Between Oct. 26, 1817, and Feb. 21, 1829? *Ans.* 11*y.* 3*mo.* 23*d.*

15. Bot. a hogshead of sugar, weighing 9*cwt.* 2*qrs.* 17*lbs.*; sold at three several times, as follows, viz: 2*cwt.* 1*qr.* 11*lbs.* 6*oz.*; 2*qrs.* 18*lbs.* 10*oz.*; 25*lbs.* 6*oz.*; what weight remains unsold? *Ans.* 6*cwt.* 1*qr.* 17*lbs.* 11*oz.*

16. A merchant receives 75 *T.* 1*qr.* 20*lbs.* of iron at one time, and 60 *T.* 15*cwt.* 3*qrs.* at another; he then sells 10 *T.* 3*qrs.* 8*oz.*; how much has he left? *Ans.* 125 *T.* 15*cwt.* 1*qr.* 19*lbs.* 8*oz.*

THEORETICAL QUESTIONS. What is compound subtraction? What the first part of the RULE?——the second?——the third?——the PROOF? What is the rule for finding the DIFFERENCE OF TIME between any two given dates?

MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS,

COMMONLY CALLED

COMPOUND MULTIPLICATION.

108. Which is a method of repeating a COMPOUND NUMBER any given number of times.

At 8*s.* 6*d.* per yard, what will 3 yards come to?

£.	s.	d.	
0	8	6,	cost of 1 yard.
		3,	number of yards.
£1	5	6,	cost of 3 yards, *Ans.*

As the cost of 1*yd.* is 8*s.* 6*d.*, it is evident, that the cost of 3*yds.* must be three times as much. We, therefore, evidently multiply the *cost of* ONE *yard by the* NUMBER *of yards*; saying 3 times 6 is 18*d.*; 18*d.* is 1*s.* and 6*d.* We set down the 6*d.*, and carry the 1*s.* to the product of the next ....saying 3 times 8 is 24, and 1 we had to carry is 25*s.*; 25*s.* is £1 5*s.* Set down the 5*s.*, and carry the £1 to the product of the next.....3 times 0 is 0, but 1 we had to carry is £1. Therefore, *Ans.* £1 5*s.* 6*d.* *This process may be used in all cases where the multiplier does* NOT EXCEED 12. Wherefore we have this

110. RULE. After placing the numbers as above, begin with the lowest, and multiply EACH denomination by the multiplier, observing to *set down* and *carry* as in COMPOUND ADDITION.

COMPOUND DIVISION.

109. Which is a method of dividing a COMPOUND NUMBER into any given number of parts.

If 3 yards cost £1 5*s.* 6*d.*, what is that per yard?

	£.	s.	d.	
3)	1	5	6,	cost of 3 yards.
Ans.	£0	8	6,	cost of 1 yard.

Here we evidently reverse the process in the left hand column, and *divide the* COST *of the given number of yards by* THAT *number*; saying 3 in £1 0 times, but £1 is 20*s.* and 5 (the number of the next less) is 25*s.*; 3 in 25 8 times, and 1*s.* over: we set down 8, and reduce the 1*s.* to the next less....saying 1*s.* is 12*d.* and 6 (the number of the next less) is 18*d.*; 3 in 18 6 times: set down 6. Therefore, *Ans.* 8*s.* 6*d.* *This process may be used in all cases where the divisor does* NOT EXCEED 12. Wherefore we have this

111. RULE. After placing the numbers as above, begin with the highest denomination, and find how often the divisor is contained in it; set the quotient underneath, reduce the remainder (if any) to the next less denomination, adding in the next less, and proceed as before.

From what has been said, it is plain, that

112. THESE TWO RULES mutually prove each other.

EXAMPLES.

1. What will 5*yds.* of cloth cost, at 14*s.* 9*d.* per yard?

2. How much will 7*yds.* come to, at 13*s.* 4*d.* per yard?

3. How much will 9*lbs.* of tea come to, at 7*s.* 6*d.* a pound?

4. If 1 silver spoon weigh 3*oz.* 11*pwt.*, how much will half a dozen weigh?

5. An apothecary would compound medicine of 8 ingredients, and put in 1 ℈ 7 ℥ 2 ℈ 19 grs. of each; of how much will this mixture consist?

6. If a bag of indigo weigh 1 *qr.* 8*lbs.* 11*oz.*, what will 11 bags weigh?

7. How many yards, &c. in 12 pieces, each containing 21*yds.* 3*qrs.* 3*na.*?

113. *If the* MULTIPLIER *is more than* 12, *and is a* COMPOSITE NUMBER, it is evident, (ART. 39,) *that we obtain the true product by observing this*

RULE. "Multiply by its COMPONENT PARTS *continually.*"

8. Frank bought 15 books, at 7*s.* 6*d.* each; how much did they come to?

The multiplier 15 = 3 × 5; therefore

£.	s.	d.	
0	7	6,	cost of 1 book.
		3,	1 of the COM. PARTS.
£1	2	6,	cost of 3 books.
		5,	the other COM. PART.

Ans. £5 12 6, cost of 15 books.

9. At 14*s.* 6*d.* a head for sheep, what will 42 cost?

10. What will 24 barrels of flour cost, at £2 12*s.* 4*d.* a barrel?

11. What will 112*lbs.* of indigo come to, at £5 14*s.* 9*d.* a pound?

12. How much brandy in 84 pipes, each containing 128*gal.* 2*qts.* 1*pt.* 3*g.*

115. *If the multiplier is more than* 12, *and is* NOT A COMPOSITE NUMBER,*

EXAMPLES.

1. If 5*yds.* of cloth cost £3 13*s.* 9*d.*, what is it per yard?

2. If 7*yds.* cost £6 1*s.* 4*d.*, what will 1 yard cost?

3. If 9*lbs.* of tea cost £5 7*s.* 6*d.*, what is it a pound?

4. If half a dozen silver spoons weigh 1*lb.* 9*oz.* 6*pwt.*, what will 1 weigh?

5. If a mixture of 15 ℈ 7 ℥ 2 ℈ 12*grs.* be compounded equally of 8 ingredients, what quantity of each is taken?

6. If 11 bags of indigo weigh 3*cwt.* 2*qrs.* 11*lbs.* 9*oz.*, what will 1 bag weigh?

7. If 12 equal pieces contain 263*yds.* 1*qr.*, what is the length of each piece?

114. *If the* DIVISOR *is more than* 12, *and is a* COMPOSITE NUMBER, *it is evident,* (ART. 54,) *that we obtain the true quotient by observing this*

RULE. "Divide by its COMPONENT PARTS *continually.*"

8. Frank bought 15 books for £5 12*s.* 6*d.*; how much was that apiece?

The divisor 15 = 5 × 3; therefore

£.	s.	d.	
5)5	12	6,	cost of 15 books.
3)1	2	6,	cost of 3 books.

Ans. £0 7 6, cost of 1 book.

9. If 42 sheep cost £30 9*s.*, what will 1 cost?

10. Bought 24 barrels of flour for £62 16*s.*; what was it a barrel?

11. If 112*lbs.* of indigo cost £642 12*s.*, what will 1 pound cost?

12. Bought 84 pipes, containing 10812*gal.* 1*qt.* 1*pt.*; how much in a pipe?

116. *If the divisor is more than* 12, *and is* NOT A COMPOSITE NUMBER,

* But if the multiplier exceed 144, the product may be more easily obtained by the *following*

RULE I. Multiply first by 10, and it will give the amount or price of TEN.

II. Multiply the amount of TEN by 10, and it will give the price of 100.

III. If the multiplier be even hundreds, multiply the amount of 100, by the number of hundreds, and it will give the answer.

IV. But if there be odd numbers, multiply the amount of 10 by the number of tens, and the amount of 1 by the number of units; then add the several products either, and it will give the product required.

Rule. Multiply by two such numbers, as, when multiplied together, come the NEAREST to the given multiplier, and the *multiplicand* by the DEFICIENCY; and add the latter product to the former.

13. Harry bought 19*yds.* of cloth, at 18*s.* 7*d.* per yard; how much did it come to?

3 × 6 = 18, and 18 + 1 = 19; therefore

£.	s.	d.	
0	18	7,	cost of 1 yard.
		6	
5	11	6,	cost of 6 yards.
		3	× 3
16	14	6,	cost of 18 yards.
0	18	7	+ 1
£17	13	1,	cost of 19 yards.

It is very evident that 6 times the cost of 1*yd.* is the cost of 6*yds.*; that 3 times the cost of 6*yds.* is the cost of 18*yds.*; and that the cost of 1*yd.* added to the cost of 18*yds.* is the cost of 19*yds.* (Art. 39.)

14. How much will 47*yds.* of cloth come to, at £2 4*s.* 6*d.* per yard?

15. How much tea in 99 chests, each 3*qrs.* 10*lbs.* 8*oz.*?

16. How much iron in 129 bars, each 3*qrs.* 3*lbs.* 3*oz.*?

Rule. Set down the numbers and *divide after the manner of* LONG DIVISION.

13. Bought 19*yds.* of cloth for £17 13*s.* 1*d.*; what was that a yard?

£.	s.	d.	£.	s.	d.
19)17	13	1	(0	18	7
20					
353*s.*					
19					
163					
152					
11*s.*					
12					
133*d.*					
133					

Proceeding after the manner of long division, 19 will not go in the £17, we therefore place 0 in the quotient under £, then reduce the *pounds* (17) to shillings, adding in the 13*s.*, and seek how often the divisor will go in the SUM 353*s.* Finding it to go 18 times, and that 11 will remain, we set 18 in the quotient under *s.*, and proceed as before. Hence the RULE is more fully expressed as follows: *Set down the numbers as in* LONG DIVISION, *divide the* HIGHEST *denomination* FIRST, *reduce the remainder to the* NEXT LESS, *add in the next less* (*if any*), *and proceed as before.*

14. Bought 47*yds.* of cloth for £104 11*s.* 6*d.*; what is that per yard?

15. How much tea must be put in a chest, to admit 84*cwt.* 1*qr.* 3*lbs.* 8*oz.* in 99?

16. If 129 bars weigh 100*cwt.* 1*qr.* 19*lbs.* 3*oz.*, what will one weigh?

THEORETICAL QUESTIONS.

What is compound multiplication? —— compound division? What is the *rule* for the MULTIPLICATION of compound numbers, when the multiplier does not exceed 12? What, for their DIVISION? What is the PROOF? When the multiplier is more

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Ex. 1. At £ 1 6*s.* 7*d.* per yard, what will 235 yards come to?

10

13	5	10,	price of 10 yds.
		10	
132	18	4,	price of 100 yds.
		2	
265	16	8,	price of 200 yds.
39	17	6,	.... 30 ..
6	12	11,	.... 5 ..

Ans. £ 312 7 1, price of 235 yds.

It is evident, that 10 times the price of 1 yd. is the price of 10 yds.; and that 10 times the price of 10 yds. is the price of 100 yds. It is likewise evident, that the price of 100 yds. × 2, is the price of 200 yds.; that the price of 10 yds. × 3, is the price of 30 yds.; that the price of 1 yd. × 5, is the price of 5 yds.; and that the price of 200 yds., 30 yds., and 5 yds., added together, is the price of 235 yds.

2. What will 359 yds. of cloth come to, at 4*s.* 7½*d.* per yard? *Ans.* £83 4½*d.*

3. In 987 barrels of flour, each containing 1 cwt. 3 qrs. 9 lbs.; How many cwt., &c.? *Ans.* 1898 cwt. 3 qrs. 7 lbs.

than 12, and is a COMPOSITE NUMBER, how do you proceed in MULTIPLICATION? How, in DIVISION? How, in multiplication, do you proceed when the multiplier is more than 12, and is NOT a composite number? How, in this case, in DIVISION?

EXAMPLES FOR PRACTICE

IN THE ARITHMETIC OF COMPOUND NUMBERS.

1. Bought 9*yds.* of Camlet, at 5*s.* 6*d.*
. . . . . 8*yds.* Flan. 3*s.* 9*d.*
. . . . . ½*yd.* Velv. 12*s.* 3*d.*
. . . . . 3 Sk. Silk, at 4*d.*
. . . . . 1 Clasp, 2*s.* 9*d.*

What is the amount? *Ans.* £4 9 4½

2. A gentleman owes to one creditor £160 2*s.* 7*d.* 1*qr.*; to another, £29 3*s.* 6*d.* 3*qrs.*; to another, £119 19*s.* 11*d.* 3*qrs.*; he has in goods, £299 19*s.* 9*d.* 3*qrs.*, and £333 13*s.* 3½*d.* in cash; how does he stand?
Ans. £623 16*s.* 11½*d. clear.*

3. A farmer sells 3 loads of hay, weighing 19*cwt.* 2*qrs.* 17*lbs.*; 16*cwt.* 16*lbs.* 15*oz.*; 17*cwt.* 3*qrs.* 1*oz.*; what is the weight of the whole?
Ans. 2 *T.* 13*cwt.* 2*qrs.* 6*lbs.*

4. Bought 3 fields containing, 17*A.* 3*R.* 16*rds.*; 25*A.* 3*R.* 19*rds.*; 11*A.* 2*R.* 25*rds.*; how much in all? *Ans.* 55*A.* 1*R.* 19*rds.*

5. A gentleman wishes to set 20 trees at equal distances in a line 18*rds.* 7*ft.* in length; how far apart must the trees be? *Ans.* 16*ft.*

6. A merchant bought 7 casks of brandy, containing as follows, viz.: 70*gal.* 3*qts.*; 67*gal.* 1*qt.*; 53½*gal.*; 75*gal.*; 101*gal.* 2*qts.*; 57*gal.* 19*qts.*; 65*gal.* 21*qts.*; how many hogsheads in the whole?

7. Bought 3 pieces of silk at Paris, measuring 31*E. Fr.* 1*na.*; 29*E. Fr.* 2*qrs.*; 28*E. Fr.* 5*qrs.* 3*na.*; of which I sold 54*yds.* 3*qrs.* 3*na.*, at the rate of 5*d.* per nail; how much more will the remainder sell for at the same rate? *Ans.* £26 7*s.* 1*d.*

8. What will 5 barrels of flour come to, at £1 10*s.* 6*d.* 3*qrs.* per barrel? *Ans.* £7 12*s.* 9*d.* 3*qrs.*

9. A silversmith has 7 tea-pots, each weighing 1*lb.* 3*oz.* 13*pwt.* 11*grs.*; what do they all weigh? *Ans.* 9*lbs.* 1*oz.* 14*pwt.* 5*grs.*

10. Suppose the earth to move through the space of 50,189,999,998*m.* 1*fur.* 20*rds.* in 18 years; how far does it move in 1 year?
Ans. 597,142,857*m.* 1*fur.* 5*rds.*

11. In £50 how many shillings, nine-pences, six-pences, four-pences, and pence, and of each an equal number? *Ans.* 31*l.*

12. A man dying, left a farm of 200*A.* 3*R.* 1*rd.*, of which his widow was to receive one-third, and the remainder to be equally divided among six children; how much must each one receive?
Ans. { The widow, 66*A.* 3*R.* 27*rds.*
Each child, 22*A.* 1*R.* 24*rds.*

13. Twenty-four men agree to construct 7*M.* 1*fur.* 24*rds.* of road; after completing ⅓ of it they employ 8 more; what distance does each man construct before and after the 8 men are employed?
Ans. 16*rds. before, and* 1*fur.* 20*rds. after.*

14. Bought 1½*lbs.* Tea . . . . . 4*s.* 6*d.* £9 7 10½
. . . . 4½ bushels corn . . . 5*s.* 4*d.* . . . . . .
. . . . 5 quarts brandy . . 8*s.* 4*d.* . . . . . .
. . . . 6 . . . . rum . . . . 7*s.* 6*d.* . . . . . .
. . . . 7½ yards Chintz . . 2*s.* 5*d.* . . . . . .

What is the amount? *Ans.* £3 11 6½

15. Bought 2 pieces muslin . . . . . 39*s*. . . . .
. . . . 25*yds*. Irish linen . . . . . 2*s*. . . . .
. . . . 28½ . . . stormount calico . . 2*s*. 6 . . .
. . . . 28¼ . . . red . . . . . 2*s*. 2 . . .
. . . . 1 piece durant . . . . . 56*s*. . . . .
. . . . 2 pieces blue shalloon . . 57*s*. 6 . . .
. . . . 50½ yards dimity . . . . . 2*s*. 6 . . .
. . . . 3 pieces persian . . . . . 84*s*. . . . .

What is the amount? *Ans.* £39 12 3

16. A cashier received one hundred pounds and nine pence half-penny, three thousand seven hundred and four pounds ten shillings, twenty thousand and ninety pounds two shillings eleven pence three farthings; he then paid away sixteen thousand sixteen hundred and sixteen pounds eleven pence three farthings; how much had he left?

Ans. £6278 12*s*. 9½*d*.

FRACTIONS.

(Continued from page 44.)

117. A fraction is said to be in its LOWEST TERMS, when its *numerator* and *denominator* are expressed by the LEAST NUMBERS possible.

From Art. 78, it is evident that we may reduce a fraction to *lower terms*, by dividing BOTH TERMS of it by any number that will divide both without a remainder; and it is also evident, that if we repeat this operation as many times as is possible, the fraction will be reduced to its LOWEST TERMS.

Ex. 1. Let $\frac{135}{180}$ be reduced to its lowest terms.

OPERATION. $5)\frac{135}{180} = \frac{27}{36} = \frac{3}{4}$. *Ans.* From the process here given, (*To reduce fractions to their lowest terms,*) we have this

118. RULE. Divide BOTH TERMS of the fraction by any number that will divide BOTH without a remainder, and the QUOTIENTS in the same manner, and so on, until there is no number that will again divide them.

2. Reduce $\frac{240}{360}$, $\frac{125}{150}$, $\frac{104}{112}$, and $\frac{840}{960}$ *respectively* to their lowest terms. *Ans.* $\frac{4}{5}$, $\frac{5}{6}$, $\frac{13}{14}$, & $\frac{7}{8}$.

119. Any number that will divide two or more other numbers without a remainder, is called a *common measure* of those numbers; and

The greatest number that will do this, is called the GREATEST COMMON MEASURE.

Thus, 3 is a common measure of 6 and 12; because it will divide them both without a remainder: $3)\frac{6}{12}(\frac{2}{4}$. But 6 is the *greatest common measure*, because it is the greatest number that will exactly divide them: $6)\frac{6}{12}(\frac{1}{2}$. Whence it is plain,

That *the Greatest Common Measure of any* TWO NUMBERS *can never exceed the* LESSER,

For, if it exceed the *lesser*, it cannot divide it, *i. e.*, it cannot be contained in it. The first step then, that is to be taken in finding the greatest common measure of two numbers, is, to to see if the lesser number, which divides or measures itself, will also divide or measure the greater. 12 in 18 goes 1 time, 6 rem.; consequently the lesser number, in this case, is not the G. C. M., because there is a *remainder*. But the number that will divide 12 & 18, must also divided the remainder 6. Between this remainder, and the lesser number (12), the same reasoning applies as above, viz., that the G. C. M. of these two, cannot exceed the lesser (6). Making this step, we find the last divisor (6) to be the common measure sought. Hence,

Ex. 1. Find the G. C. M. of $\frac{12}{18}$.

12)18(1
12
—
6)12(2

120. *To find the* GREATEST COMMON MEASURE *of two numbers,*

RULE. Divide the GREATER NUMBER by the LESS, and THAT DIVISOR by the REMAINDER, and so on, until nothing remain; the LAST DIVISOR used will be the GREATEST COMMON MEASURE.

NOTE 1. If we divide both terms of a fraction, by their G. C. M. we shall evidently obtain the fraction in its *lowest terms*.

Ex. 2. Find the G. C. M. of $\frac{45}{63}$, and by it reduce the fraction to its lowest terms.

45)63(1
45
—
18)45(2
36
—
9)18(2
18
—
0

Then $9)\frac{45}{63}(\frac{5}{7}$, *Ans.*

NOTE 2. The G. C. M. of *more than two* numbers may be found by finding the G. C. M. of TWO numbers, and then of THAT common measure, and ONE *of the other numbers*, and so on, to the last.

3. When the numbers by which the terms of a fraction are divisible, do not readily appear, the method by the G. C. M. is to be preferred.

Ex. 3. Reduce $\frac{91}{117}$, $\frac{114}{285}$, $\frac{144}{216}$, $\frac{192}{576}$, and $\frac{3619}{6251}$ *respectively* to their lowest terms. *Ans.* $\frac{7}{9}$, $\frac{2}{5}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{11}{11}$.

MULTIPLY $\frac{1}{2}$ BY $\frac{2}{3}$.

The multiplicand $\frac{1}{2}$ is $\times \frac{2}{3}$, by dividing it by 3, and multiplying the quotient by 2. (Art. 80.) First then, $\frac{1}{2}$ is $\div$ 3, thus $\frac{1}{2} \times 3 = \frac{1}{6}$ (Art. 87 & 77); and $\frac{1}{6}$ is $\times$ 2, thus, $\frac{1}{6} \times 2 = \frac{2}{6}$. (Art. 81 & 76). But these two operations may be performed at once, thus, $\frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$, *Ans.*

This principle may evidently be extended so as to apply to the multiplication of *more than two* fractions into one another. Hence,

121. *To multiply* FRACTIONS *by* FRACTIONS,

RULE. Multiply the NUMERATORS together for a NEW NUMERATOR, and the DENOMINATORS together for a NEW DENOMINATOR.

Ex. 1. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60} = \frac{2}{5}$. 2. $\frac{3}{5} \times \frac{5}{7} \times \frac{7}{8} = \frac{105}{280} = \frac{3}{8}$.

122. A COMPOUND FRACTION is a fraction OF a fraction.

It is known by the word OF, as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{1}{2}$, $\frac{1}{5}$ of $\frac{4}{5}$ of $\frac{5}{6}$, &c.

The word OF *in this case implies* MULTIPLICATION.

As $\frac{1}{2}$ of $\frac{2}{3}$ implies $\frac{1}{2} \times \frac{2}{3}$, and $\frac{3}{4}$ of $\frac{1}{2}$ implies $\frac{3}{4} \times \frac{1}{2}$, and so on.

122 *a.* But according to principles in Arts. 78 & 118, the operation $\frac{1}{2} \times \frac{4}{5}$ may be shortened; for the several numerators and *denomi-*

nators are FACTORS of the NEW numerator and denominator, and by omitting *those which are the same in* BOTH TERMS, *we obtain the* NEW FRACTION *in lower terms.*

Thus, with the above, omitting or *cancelling* the terms that are found both among the numerators and denominators, we have

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{2}{5}, \textit{Ans.}$$

122 *b.* Upon the same principle, (Art. 78,) *if any of the numerators and denominators be divisible by the* SAME *number, the division may be executed before multiplying, and the* NEW FRACTION *will be obtained in still lower terms.*

EX. 3. What is $\frac{2}{3}$ of $\frac{10}{12}$ of $\frac{7}{15}$ = ?

2 will divide itself, and it will also divide 12; 5 will divide 10, and it will also divide 15; hence

$$\frac{\cancel{2}}{3} \text{ of } \frac{\cancel{10}}{\cancel{12}} \text{ of } \frac{7}{\cancel{15}} = \frac{\cancel{21}}{3} \text{ of } \frac{\cancel{10}2}{\cancel{12}6} \text{ of } \frac{7}{\cancel{15}3} = \frac{14}{54} = \frac{7}{27}, \textit{Ans.}$$

4. $\frac{2}{7}$ of $3\frac{2}{3}$ of $\frac{3}{4}$ of 9 = how much?

NOTE. *Mixed numbers* must be reduced to *improper fractions*, and whole numbers to the *form of fractions*. *Ans. to the last,* $\frac{99}{14}$.

5. How much is $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{20}$? *Ans.* $\frac{7}{48}$.

6. What cost $12\frac{3}{4}$*yds.* at $\frac{3}{8}$ of a $ pr. yard? *Ans.* $\$4\frac{51}{32}$.

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7. What is the value of $\frac{3}{4}$ of a £?

122 *c.* The principle here involved, is called *finding the value of a fraction in the known parts of the integer.* The integer here is £1, and its known parts 20*s*. The question therefore may be presented in this way: What is the value of $\frac{3}{4}$ of 20*s*., or $\frac{3}{4}$ of $\frac{20}{1}$?

$\frac{3}{4}$ of $\frac{20}{1} = \frac{60}{4} = 13s.\ 4d.$, *Ans.*

8. What is the value of $\frac{3}{5}$ of a *lb.* Troy? *Ans. 7oz. 4pwt.*

9. What is the *value of* $\frac{1}{8}$ *of a degree? Ans. 60m. 6fur. 20rds.*

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10. What part of a £ is 13*s*. 4*d*.?

122 *d.* The principle here involved is the *reverse* of the foregoing, and is generally called *reducing a given quantity to the fraction of a greater denomination of the same kind.* 13*s*. 4*d*. = 160*d*., £1 = 20*s*. = 240*d*.; therefore 1 penny would be $\frac{1}{240}$ of a £., and 160*d*. would be $\frac{160}{240} = \frac{2}{3}$, *Ans.*

11. Reduce 7*oz*. 4*pwt*. to the fraction of a *lb*.

12. Reduce 60*m*. 6*fur*. 20*rds*. to the fraction of a degree.

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13. Reduce $\frac{3}{4}$ of a penny to the fraction of a shilling.

122 *e.* The principle involved in this question is that which is commonly called *reducing a fraction of one denomination to the fraction of* A GREATER, *retaining the same value.*

$\frac{3}{4}$ of a penny is $\frac{3}{4}$ of $\frac{1}{12}$ of a shilling = $\frac{3}{48}$ or $\frac{1}{16}$ of a shilling, *Ans.*

14. Reduce $\frac{4}{7}$ of a *lb*. Avoir. to the fraction of a cwt. *Ans.* $\frac{1}{196}$.

15. What part of 3*d*. is $\frac{2}{3}$ of 2*d*.

16. Reduce $\frac{3}{4}$ of a grain Troy, to the fraction of a *lb*. *Ans.* $\frac{1}{5760}$.

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17. Reduce $\frac{1}{16}$ of a shilling to the fraction of a penny.

122 *f.* The principle here involved is the reverse of the preceding, and

is commonly called *reducing a fraction of one denomination to the fraction of* A LESS, *retaining the same value.* $\frac{1}{16}$ of a shilling is $\frac{1}{16}$ of 12*d.* $= \frac{1}{16}$ of $\frac{12}{1} = \frac{12}{16} = \frac{3}{4}$, *Ans.*

18. Reduce $\frac{1}{100}$ of a *cwt.* to the fraction of a *lb.*

19. Reduce $\frac{1}{320}$ of a *lb.* Troy to the fraction of a grain.

20. Reduce $\frac{1}{8}\frac{1}{160}$ of a mile to the fraction of a foot.

DIVIDE $\frac{2}{3}$ BY $\frac{3}{4}$.

The dividend is divided by $\frac{3}{4}$ by multiplying it by 4, and dividing the product by 3, (Art. 86.) $\frac{2}{3}$ is multiplied by 4, thus $\frac{2}{3} \times 4 = \frac{8}{3}$, (Arts. 81 and 76); and $\frac{8}{3}$ is divided by 3, thus $\frac{8}{3} \times \frac{1}{3} = \frac{8}{9}$, *Ans.*

The process, it will be seen, consists in *multiplying the* NUM. *of the dividend by the* DENOM. *of the divisor, and the* DENOM. *of the dividend by the* NUM. *of the divisor.* Hence,

123. *To divide* ONE *fraction by* ANOTHER,

RULE. INVERT THE DIVISOR AND PROCEED AS IN MULTIPLICATION.

Ex. 1. Divide $\frac{7}{8}$ by $\frac{3}{5}$. $\frac{7}{8} \times \frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}$, *Ans.*

2. Divide $\frac{11}{12}$ by $\frac{4}{9}$. *Ans.* $2\frac{1}{16}$.

3. Divide $\frac{13}{14}$ by $\frac{9}{1}$. *Ans.* $1\frac{4}{14}$.

4. Divide $\frac{2}{3}$ of $\frac{7}{8}$ by $\frac{3}{4}$ of $\frac{1}{10}$. *Ans.* $\frac{70}{81}$.

5. Divide $\frac{2}{3}$ of $\frac{9}{9}$ of a £. by $\frac{4}{5}$ of $\frac{7}{8}$ of a $. *Ans.* $2\frac{22}{160}$*s.*

ADDITION AND SUBTRACTION.

124. Before fractions can be added or subtracted they must be reduced to parts of the same magnitude.

William had a fine large melon; he gave $\frac{2}{8}$ of it to his sister Jane, and $\frac{1}{8}$ to little Mary: How much of the melon did he give away? How much did he keep?

$\frac{2}{8} + \frac{1}{8}$ how much? $\frac{8}{8} - (\frac{2}{8} + \frac{1}{8})$ how much?

It is plain that *fractions of the same magnitude or denominator, may be added or subtracted by adding or subtracting their* NUMERATORS.

$\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$, the quantity given away; and $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$, the quantity left.

125. Edwin has $\frac{2}{3}$ of an apple, his brother John $\frac{3}{5}$; how much had they both? —— how much more had the one than the other?

Here, the parts being of *different* magnitudes, must first be reduced to the SAME, that is, to a *common denominator.* If the 3ds. be each divided into 5 equal parts, and the 5ths. each into 3 equal parts, they will both become 15ths. The $\frac{2}{3}$ will become $\frac{10}{15}$, and the $\frac{3}{5}$ $\frac{9}{15}$; then $\frac{10}{15} + \frac{9}{15} = \frac{19}{15}$, the sum which they both had. And $\frac{10}{15} - \frac{9}{15} = \frac{1}{15}$, which Edwin had more than John.

OPERATION. $\frac{2}{3} + \frac{3}{5} = \frac{2 \times 5}{3 \times 5} + \frac{3 \times 3}{3 \times 5} = \frac{10}{15} + \frac{9}{15} = \frac{19}{15}$

Ex. 2. Execute $\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$ .... $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{2 \times 3 \times 5}{3 \times 4 \times 5} + \frac{3 \times 3 \times 5}{3 \times 4 \times 5} + \frac{4 \times 3 \times 4}{3 \times 4 \times 5} = \frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = 2\frac{13}{60}$, *Ans.*

By multiplying *all the denominators* together for a COMMON *denominator*, as in this and the previous example, we evidently obtain a number of which each denominator is a FACTOR; and by multiplying each numerator into all the denominators EXCEPT ITS OWN, we multiply it by the *same number*: consequently the value of the fractions will not be altered. (See Art. 78.) Hence,

126. *To reduce fractions of different denominators, to equivalent fractions, having a* COMMON DENOMINATOR, *we have this*

RULE. Multiply *all the denominators* together for a COMMON DENOMINATOR, and *each numerator* into all the denominators EXCEPT ITS OWN for a NEW NUMERATOR.

3. Reduce the fractions $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{5}{7}$, to a common denominator.

$4 \times 5 \times 7 = 140$, *common denominator.*

$3 \times 5 \times 7 = 105$
$4 \times 4 \times 7 = 112$ } *new numerators.*
$5 \times 4 \times 5 = 100$

The new fractions therefore, are $\frac{105}{140}$, $\frac{112}{140}$, and $\frac{100}{140}$.

4. Reduce $\frac{3}{7}$, $\frac{5}{8}$, and $\frac{9}{10}$, to fractions having a common denominator.

Ans. $\frac{240}{560}$, $\frac{392}{560}$, *and* $\frac{504}{560}$.

5. Add together $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{17}{24}$.

6. A boy carried berries to market; he sold $\frac{1}{2}$ of what he had to one person, and $\frac{1}{3}$ of what he then had to another; how much had he left?

Ans. $\frac{1}{3}$.

7. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to a common denominator.

Ans. $\frac{3}{6}$ & $\frac{4}{6}$, *or* $\frac{9}{18}$ & $\frac{12}{18}$.

From the operations, &c. above, it is evident, that the *common denominator* may be any number, of which each *denominator* is a FACTOR; for the numerators may evidently be increased accordingly. See Art. 118.

127. Any number that is divisible by *two or more other numbers* without a remainder, is called a *common multiple* of those numbers; and

The least number that is capable of this, is called the LEAST COMMON MULTIPLE.

Hence, the *least common denominator* of any fractions is the least *common multiple of all their denominators.* By the rule already given we always find a *common multiple* of the given denominators, but we do not always find the *least common multiple.* In the last example, 24 is a common multiple of 4 and 6, but it is not their *least* common multiple; for 12 is susceptible of the same. In order to explain the method of finding the least common multiple, we shall propose the following

EX. Find the least common multiple of 4 and 6.

It is evident, from our definition, that *no number can be a multiple of another, unless it contain* ALL THE FACTORS *of that other number.*

The factors of 4 are	. . . .	2 & 2
Do. of 6 "	. . . .	2 & 3

But $2 \times 2 \times 3 = 12$ contains the factors of 4, and also of 6. Therefore 12 is a common multiple of 4 & 6, and it is their *least* common multiple, because it does not contain any factor more than is absolutely necessary to produce them. The factor 2 has been once omitted, because it was common to both the given numbers, and it may be observed IN GENERAL, that

5. If I, with 25 others, build $\frac{3}{4}$ of a mile of wall, what part of it do I build? *Ans.* $\frac{3}{104}$

6. A man bought 3 horses for \$376$\frac{3}{17}$, and 5 sheep for $\frac{3}{10}$ of the value of one horse; what was the average price of the horses, and what the cost of 1 sheep? ——— 5 sheep? *Ans. to last,* \$32$\frac{102}{1010}$

7. A man owning $\frac{3}{11}$ of a share in a certain canal stock, sold $\frac{2}{17}$ of his part, for £39 17s. 6d.; what is the value of a share?

Ans. £1244 11s. 1$\frac{13}{24}$d.

8. What costs a job of work performed by 3 men, if the first man perform $\frac{1}{2}\frac{2}{5}$; the second $\frac{1}{10}$; and the third the remainder, for which he receives \$57? *Ans.* \$2243$\frac{5}{14}$

9. A post is $\frac{1}{6}$ in the ground, $\frac{2}{7}$ in the water, and 17 feet out of water; what is its whole length? *Ans.* 31$\frac{1}{42}$ *feet.*

10. There are 3 pieces of cloth, one containing 7$\frac{1}{4}$ yards, another 13$\frac{2}{3}$ yards, and the other 15$\frac{1}{2}$ yards; how much in the whole?

11. A and B can do a piece of work in 8 days, A can do it in 13 days; in what time can B do it? *Ans.* 20$\frac{4}{5}$ *days.*

12. If several men own equal shares, and a boy the overplus, of $\frac{2}{11}$ of a ship's cargo, and each man's share is $\frac{3}{10}$ of the whole; how many owners to the $\frac{9}{11}$? and, if each man's share be worth \$599, what is the value of the whole cargo? *Ans.* 5 *owners to the* $\frac{9}{11}$ *besides the boy, whose share is* \$108$\frac{10}{11}$*; the whole cargo is worth* \$3793$\frac{1}{2}$.

13. A certain school house is owned by 14 proprietors, and rented for \$24 annually. A and B own each $\frac{1}{15}$ of the house; C $\frac{2}{15}$; D and E each $\frac{1}{24}$; F $\frac{1}{12}$; G $\frac{1}{24}$ and $\frac{1}{2}$ of $\frac{1}{24}$; H $\frac{1}{15}$ and $\frac{1}{4}$ of $\frac{1}{15}$; K $\frac{1}{15}$ and $\frac{1}{2}$ of $\frac{1}{12}$; L $\frac{3}{4}$ of $\frac{1}{15}$; M $\frac{3}{4}$ of $\frac{1}{24}$; N $\frac{23}{480}$; O $\frac{1}{24}$ and $\frac{1}{2}$ of $\frac{1}{15}$; P $\frac{1}{15}$ and $\frac{3}{4}$ of $\frac{1}{15}$. What ought each proprietor to receive annually for his share of the rent?

Ans. A & B each \$1$\frac{3}{5}$*; C* \$3$\frac{1}{5}$*; D & E each* \$1*; F & H each* \$2*; G* \$1$\frac{1}{2}$*; K* \$1$\frac{2}{5}$*; L* \$1$\frac{1}{5}$*; M* \$0$\frac{3}{4}$*; N* \$1$\frac{3}{20}$*; O* \$1$\frac{4}{5}$*; and P* \$2$\frac{4}{5}$.

DECIMAL FRACTIONS.*

131. We have seen, Art. 13, that "each removal of place towards the *left hand* INCREASES the value of a figure TEN TIMES." The contrary is therefore evidently true, viz., that *each removal of place towards the right hand* DECREASES *the value of a figure also* TEN TIMES.

132. We may evidently extend this reasoning below the INTEGRAL UNIT; and obtain for each successive place towards the right hand a value *ten times smaller* than that immediately preceding it.

Hence we shall have for the first place to the right hand of units, *tenth* parts ($\frac{1}{10}$); for the second place, *hundredth* parts ($\frac{1}{100}$); for the third place, *thousandth* parts ($\frac{1}{1000}$); and so on.† These and such like expressions, are called

333. DECIMAL FRACTIONS; *in which an* INTEGRAL UNIT *is supposed to be divided into* TEN *equal parts, and each of these into ten* OTHER *equal parts,* and so on.

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*al is derived from the Latin *decem*, which signifies TEN.

†The figure of ANY PLACE whatever, may evidently be considered as in the preceding place.

134. Hence the denominator to a decimal fraction will always

	&c.	Hundreds,	Tens,	Units,		Tenths,	Hundredths,	Thousandths,	&c.	
EXAMPLE.		2	2	2	.	2	2	2	=	$222\frac{222}{1000}$
			3	3	.	3	3		=	$33\frac{33}{100}$
				4	.	4			=	$4\frac{4}{10}$

Be 10, 100, 1000, &c.; or 1 *with as many ciphers annexed* as the NUMERATOR HAS PLACES.

135. Consequently the DENOMINATOR to a decimal need not be expressed, for the NUMERATOR *with a point* (.) *before it*, will alone be sufficient to determine the value.

.7854 is a decimal consisting of four places; its denominator is therefore 1 *with four ciphers annexed:* thus $\frac{7854}{10000}$.

136. Any DECIMAL may be expressed in the form of a *vulgar fraction*, by placing under it, its PROPER DENOMINATOR.

Thus .325 may be expressed $\frac{325}{1000}$; .7854, $\frac{7854}{10000}$, &c.

NOTE. Numbers consisting of INTEGERS and DECIMALS are called MIXED DECIMALS.

TABLE.

| | | | |

137. As it appears from the table

We may evidently consider UNITS as a *starting point*, from which INTEGERS proceed continually *increasing* in a tenfold relation towards the left hand, and DECIMALS continually *decreasing* in the same relation towards the right.

138. *The value of* EVERY FIGURE *is determined by its place from* UNITS. Consequently;

139. CIPHERS placed at the *right hand* of decimals do NOT alter their value.

Thus, .5, .50, .500, are all of the same value, each being = .5 or $\frac{1}{2}$.

140. But EVERY cipher, which is placed at the *left hand* of decimals DECREASES *their value tenfold.*

Thus, .5, .05, .005, each express a different value; .5 being equal to $\frac{5}{10}$ or $\frac{1}{2}$; .05 to $\frac{5}{100}$ or $\frac{1}{20}$; .005 to $\frac{5}{1000}$ or $\frac{1}{200}$.

141. Decimal fractions, having DIFFERENT DENOMINATORS are reduced to those of the SAME by annexing ciphers until the *number of places* become EQUAL.

Thus, .3, .04, .025, are reduced to .300, .040, .025, each having 1000 for a *common denominator.*

142. Decimals are read the same as whole numbers, giving the name of the lowest or right hand place, to the WHOLE.

Thus, .5236 is read 5236 *ten thousandths*, or $\frac{5236}{10000}$.

143. ANY WHOLE NUMBER may be reduced to *decimal parts* by annexing ciphers.

25 is reduced to *tenths* by annexing a cipher, to *hundredths* by annexing two ciphers, &c. Thus 25 = 250 tenths = 2500 hundredths, &c.

144. ANY MIXED DECIMAL may be read TOGETHER by giving to it the name of the *lowest place.*

Thus, 63.25 may be read 6325 *hundredths*, and the whole may be expressed in the form of a vulgar fraction: $\frac{6325}{100}$.

FEDERAL MONEY

145. IS THE COIN of the UNITED STATES.

The denominations are made to correspond to the decimal divisions now described; the dollar being the UNIT, dimes *tenths*, cents *hundredths*; and so on.

146. As the denominations of this money correspond, in value, to the orders of simple numbers, it follows that they may be *read* the same; that is, either *wholly* in the LOWEST DENOMINATION, or *partly* in the LOWEST, and *partly* in a HIGHER.

Eagles	dollars	dimes	cents	mills
3	2	4	5	6,

Thus: 3 2 4 5 6, may be read 39456 *mills*, or 3945 *cents* and 6 *mills*, or (if we reckon the eagles *tens of dollars*, and the dimes *tens of cents*,) the [illegible] may be read 32 *dollars*, 45 *cents*, and 6 *mills*.

[illegible]pressions in *dollars*, *cents*, and *mills*, form what we have cal[illegible] DECIMALS.

Thus, the above sum forms the *mixed decimal* $32.25, that is, *dollars* and *decimals of a dollar*.*

ADDITION AND SUBTRACTION OF DECIMALS.

1. What is the amount of 7.05, and 16.723?

Tens.	Units.	Tenths.	Hundredths.	Thousandths.
	7	.	0	5 0
1	6	.	7	2 3
2	3	.	7	7 3

Considering the whole of each number as so many *thousandths*, (Art. 142,) we have $\frac{7050}{1000} + \frac{16723}{1000} = \frac{23773}{1000}$, the amount required. But $\frac{23773}{1000} = \$23.773$, *Ans.* The 0 is placed at the right hand of .05, to reduce it to the denominator of the other decimal; but this may as well be understood.

2. Bought a cow for $17.25, paid $9.875; how much remains?

$17.25	Considering the whole of each number as so many *mills* or *thousandths*
9.875	of a dollar, (Arts. 142, 144,) we have $\frac{17250}{1000} - \frac{9875}{1000} = \frac{7375}{1000} =$
$ 7.375	$7.375, *Ans.*

148. Hence, *to* ADD *or* SUBTRACT DECIMALS, we have this

RULE. Place the numbers so that the decimal points may stand under each other; then ADD or SUBTRACT as in simple numbers, and place the decimal point in the result, directly under those of the given numbers.

EXAMPLES.

3. Bought camlet $4, flannel $3.75, two skeins of silk $0.12½, one clasp $1.23; how much in all?

$4.	=	4000 *mills*, or 1000ths of a dollar.
3.75	=	3750 *mills*, or 1000ths ... .
.125	=	125 *mills*, or 1000ths ... .
1.23	=	1230 *mills*, or 1000ths ... .
Ans. $9.105	=	9105 *mills*, or 1000ths ... .

4. Borrowed $315, paid $115.75; how much remains?

$315.	=	31500 *cents*, or 100ths of a dollar.
115.75	=	11575 *cents*, or 100ths ... .
Ans. $199.25	=	19925 *cents*, or 100ths ... .

5. What is the amount of $113.25, $70.005, $6.875, $1750.4, and $19.125? *Ans.* $1959.655
6. ——— of $2.1, $0.5, $26.177, $0.7, and $5? *Ans.* 34.477
7. ——— of $42.23, $9.4, $18.47, $52.384, $305.105, $29.003, & $34.19? *Ans.* $490.782
8. From $5.83 take $4.2753001. *Ans.* $1.5506999
9. From 480 take 245.0075. *Ans.* 234.9925
10. A man, owing $4316, paid $376.865; how much did he then owe? *Ans.* $3939.135

* For the denominations, &c. of federal money, see page 54.

11. What is the sum of twenty-nine and three tenths; three hundred and seventy-four, and nine millionths; ninety-seven, and two hundred and fifty-three thousandths; three hundred and fifteen, and four hundredths; twenty-seven; one hundred and four tenths? *Ans.* 942.993009

MULTIPLICATION OF DECIMALS.

1. Bot. 7 head of hogs at $3.25 a head; how much did they come to?

Operation.

3.25
7

$\frac{2275}{100} = \$22.75$, *Ans.*

Ans. $22.75

Regarding the multiplicand as so many *cents* or *hundredths* of a dollar, with the denominator expressed, we have $\frac{325}{100} \times 7$

2. How much will .75*yd.* come to at $ .25 per yard?

.75
.25

375
150

Ans. .1875

If we regard the factors as vulgar fractions, we shall have $\frac{75}{100} \times \frac{25}{100} = \frac{1875}{10000} = .1875$. Hence, *there must always be* AS MANY *places pointed off for decimals in the* PRODUCT, *as there are decimal places in* BOTH FACTORS.

3. Mult. .127 by .04.

.127
.04

Ans. .00508

From the foregoing it follows, that when the product does not give so many places as there are decimals in both factors, *the* DEFICIENCY *must be supplied by prefixing ciphers.* $.127 = \frac{127}{1000}$, and $.04 = \frac{4}{100}$. Then $\frac{127}{1000} \times \frac{4}{100} = \frac{508}{100000} = .00508$, *Ans.*

Hence *for the* MULTIPLICATION OF DECIMALS, we have this

149. RULE.* Multiply as in whole numbers, and from the product point off SO MANY PLACES for decimals, as there are decimal places in BOTH FACTORS; but if there are not so many places, supply the deficiency by prefixing ciphers.

EXAMPLES.

4. What cost 13*yds.* at $7.625 per yard? *Ans.* $99.125.
5. At $ .1875 a pound for butter, what cost 7.625 pounds. *Ans.* $1.429 +
6. Required the product of .0053 and .00007. *Ans.* .000000371.
7. How much will 93*yds.* of shalloon come to at $ .53 a yard? *Ans.* $49.29.
8. At 21 cents 9 mills a pound for coffee, what cost 187*lbs.* *Ans.* $40.953.
9. Bought 17 horses, at $58.39 a head; how much did they come to? *Ans.* $992.63.
10. At $ .09 a pound, what cost 187.5 pounds of sugar? *Ans.* $16.875.
11. If one day's labour be $1.62½ what will 314 day's amount to? *Ans.* $510.25.
12. What will 36*lbs.* of flax come to at $ .17 a pound? *Ans.* 6.12.
13. At $ .59 a bushel for corn, what will 64 bushels come to? *Ans.* $37.76.
14. What will 314.96*ft.* of boards come to at $8. per M.? *Ans.* 251.968

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*ply a decimal by 10, 100, 1000, &c. we have only to remove the separ- places towards the right hand as there are ciphers in the multiplier.

150. In selling goods, it is usual for the seller to deliver to the buyer a bill of the articles bought, &c. Such bills are sometimes called

BILLS OF PARCELS.

New-York, Aug. 14, 1828.

Mr. James Paywell

Bought of Simeon Thrifty

3 hhd. new Rum, 118*gal.* each,	at	$ 0.31	per	gal.
2 pipes French Brandy, 126 & 132*gal.*	"	1.12½	"	"
1 hhd. brown Sugar, 9¾*cwt.*	"	10.34	"	cwt.
3 casks of Rice, 2*cwt.* 1*qr.* 17*lbs.* each,	"	.05	"	lb.
5 bags Coffee, 75*lbs.* each,	"	.23	"	"
1 chest Hyson Tea, 86*lbs.*,	"	.92	"	"

Received payment, $706.52½

For Simeon Thrifty,

PETER FAITHFUL.

Wilderness, Feb. 8, 1827.

Mr. Peter Carpenter

Bought of Asa Falltree

5682	feet Boards,	at $	6.	per	M.
2000	" "	"	8.34	"	"
800	" Thick Stuff,	"	12.64	"	"
1500	" Lathing,	"	4.	"	"
650	" Plank,	"	10.	"	C.
879	" Timber,	"	2.50	"	"
236	" "	"	2.75	"	"

Received payment, $101.849

ASA FALLTREE.

NOTE. M. stands for the Latin *mille*, which signifies 1000, and C. for *centum*, which signifies 100.

DIVISION OF DECIMALS.

151. We have seen, in multiplication, that the decimal places in the product must always be equal to the number of decimal places in *both factors*. The *multiplicand* and *multiplier*, become the *divisor* and *quotient* in division. Hence it follows, that in DIVISION OF DECIMALS, *the number of places* in the DIVISOR AND QUOTIENT, *counted together, must always be* EQUAL *to the number of places in the* DIVIDEND.

1. If 7 head of hogs cost $22.75, what is the cost of one?

7)22.75 | Regarding the dividend, $22.75, as a vulgar fraction, we have $\frac{2275}{100} \div 7$, (*Arts.* 86 & 76.) $= \frac{325}{100} =$ $3.25, *Ans.* Consequently, *when the* DIVISOR *has* NO *decimal places, the* QUOTIENT *must have just* AS MANY *as there are decimal places in the* DIVIDEND.

Ans. $ 3.25

2. At $6.75 a yard for cloth, how many yards may be bought for $53?

Regarding the divisor $6.75, as a vulgar fraction, we have $53 \div \frac{675}{100}$ (Art. 86) $= \overline{53 \times 100} \div 675 = \frac{5300}{100} \div \frac{675}{100} = 6.75)53.00$ (

Hence, if the DIVIDEND has not so many decimal places as the DIVISOR-

it must be MADE to have AS MANY, (by annexing ciphers,) before the DIVISION can be commenced.

Making this preparation with the above, we have

6.75) 53.00 (7.85 + Or 7yds. & 85 hds. Ans.
47 25
5750
5400
3500
3375
125

From what has been already said above, it will be easy to see, that we shall *continue to have* WHOLE NUMBERS, *until as many decimal places of the* DIVIDEND *are* USED *as there are decimal places in the* DIVISOR. Hence it will be convenient in practice *to count off the number of decimal places* in the *divisor*, *and as many in the dividend, and make a point over the last;* and then, when the figure is brought down over which this point is placed, and the quotient figure from the number which it makes is obtained, *we may* ALWAYS *observe to place a point in the quotient:* and after this the division may evidently be extended to decimal parts, by annexing a cipher to each subsequent remainder, by which is effected the reduction of it to TENTH parts of the preceding, and consequently each resulting quotient figure will represent *tenth* parts of the preceding one.

Now, as every cipher annexed to the remainders must be accounted a decimal of the dividend, we may observe, that the pointing as above, agrees exactly with the principle first above given. There are *four* decimal places in the dividend, and but *two* in the divisor; therefore there should be *two* in the quotient, which is the case.

3. Divide .00508 by .127

.127).00508 (.04
508

Pointing the dividend as above, we find that the divisor will not go in the same number of decimal places of the dividend; therefore there will be no *integers*. Making a point in the quotient, and taking in another figure of the dividend, (thereby reducing it to *tenth* parts of what we took before) we find again that the divisor will not go, therefore there will be no *tenths*. Placing 0 in the quotient, and taking *another* figure of the dividend, we find that the divisor will go 4 times *precisely*. Therefore the quotient is .04

Considering the principle first above, we find it to apply equally well here. For, in this instance, there are *five* decimals in the dividend, and *three* in the divisor; therefore there must be *two* in the quotient, and as the division gives but one, we must supply the *deficiency* by prefixing a cipher.

152. Hence *for the* DIVISION OF DECIMALS, we have this

RULE.* Divide as in whole numbers, and point off from the quotient as many places for decimals, as the decimal places in the dividend exceed those in the divisor; but if there are not so many places, supply the deficiency by prefixing ciphers.

NOTE. The quotient may be carried to any degree of exactness, by annexing ciphers to the remainders, and continuing the division, but the ciphers annexed must be accounted so many decimals of the dividend.

EXAMPLES.

4. If 798 quintals of fish cost $2840.88, what is it per quintal?
Ans. $3.56

5. Bought a farm, containing 149 acres, for $9000.; what did it cost per acre? *Ans.* $60.3355 +

6. Sold 750 pounds of wool for $468.75, what did I get a pound?
Ans. $ .625

* As dividing by 10, 100, 1000, &c., we have only to remove the separatrix to the right hand as there are ciphers, so in dividing by the decimal, remove the separatrix in the same manner to the *left*.

7. If 3 yards of cloth cost $5.25, what is it a yard? *Ans.* $1.75
8. Bought 18 gallons of brandy, for $42.75; what was that per gallon? *Ans.* $2.375
9. If 205 yards of durant cost $107.62½, what is it per yard? *Ans.* $ .52½
10. Bought 125 gallons of molasses, for $95; what did it cost per gallon? *Ans.* $ .76
11. A ship's company take a prize worth $12465; of which the owner takes ½, the officers ⅛, and the remainder is equally divided among the sailors, who are 125 in number; how much is each sailor's part? *Ans.* $24.93
12. If a yard of cassimere cost $2.31, how many yards can be bought for $50.623? *Ans.* 21.91+ *yards.*
13. The salary of the President is $25,000 a year; how much is that per day? *Ans.* $68.493+
14. A owes B $15.58, and is to pay him in rye at 67 cents a bushel; how much rye will it require to discharge the debt? *Ans.* 23.25*B.*+
15. A privateer having taken a prize worth $6850, it is divided into 100 shares; of which the captain is to have 11; two lieutenants, each 5; 12 midshipmen, each 2; and the remainder is to be divided equally among the sailors, who are 105 in number; what must each one receive?
Ans. Capt. $753.50, *Lieut.* $342.50, *Midshipman* $137, *Sailor* $35.88

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

1. Reduce ¾ of a dollar to decimal parts.

As the *numerator* is the DIVIDEND in division, and the *denominator* the DIVISOR the above may be presented in this way:

```
4)30 (.75
  28
  --
  20
  20
  --
4 | 3
  |---
  | .75
```

It is plain that there can be no integers, because the fraction is less than a unit. We therefore place a point in the quotient, and reduce the dividend to TENTHS by annexing a cipher to it (Art. 151, Ex. 2 and 3); and obtain for a quotient figure .7, and 2 *tenths* remainder. Annexing a cipher to this remainder, we reduce it to *tenths of tenths*, that is, to *hundredths*; and dividing we obtain 5 *hundredths* and have no remainder. The decimal expression of ¾ is therefore .75.

This process may evidently be executed by short division, and a cipher *supposed* to be annexed every time when there is a remainder, and the division continued as far as may be necessary.

153. Hence, *To reduce a* VULGAR *fraction* to a DECIMAL, we have this

RULE. Divide the *numerator* by the *denominator* as in DIVISION OF DECIMALS, and the quotient will be the decimal required.

EXAMPLES.

2. Reduce ⅛ to a decimal.

```
8 | 1.
  |---
  | .125, Ans.
```

3. Reduce ½, ⅓, ¼, and ⅕ to decimals.
Ans. .25, .5, .8, .875.

4. Reduce $\frac{4}{66}$, $\frac{9}{1120}$, and $\frac{11}{6666}$ to decimals.
Ans, .0606+*, .00797+, and .000183+

* 'Decimal figures, which *continually repeat*, like .06, in this example, are called *Repetends* or *Circulating* Decimals. If only *one figure* repeats, as .3333 or .7777 &c., it is called a *single repetend*. If *two* or *more figures* circulate alternately, as .565656

5. Reduce $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$, $\frac{1}{99}$, $\frac{5}{99}$, $\frac{1}{999}$, $\frac{7}{999}$, $\frac{1}{9999}$, $\frac{8}{9999}$ to decimals.
Ans. .3̇, .6̇, .1̇1̇, .2̇2̇, .4̇4̇, .0101̇, .0505̇, .00110011̇, .007007̇, .001001̇, .008008̇.

REDUCTION OF COMPOUND NUMBERS TO DECIMALS.

154. *To reduce a compound number to the decimal of a higher denomination.*

1. Reduce 7*s.* 6*d.* to the decimal of a pound.

155. *To reduce the decimal of a higher denomination to its* VALUE.

1. Reduce £.375 to its value.

.375 of a pound $= \frac{375}{1000}$ of 20*s.* $= \frac{375}{1000}$ of $\frac{20}{1} = \frac{7500}{1000} = 7.500$*s.*, *i. e.*,

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.234234234 &c., it is called a *compound repetend.* If other figures arise *before* those which circulate, as .743333, .143010101 &c., the decimal is called a *mixed repetend.*

A single repetend is denoted by writing only the *circulating figure* with a point over it: thus, .3̇, signifies that the 3 is to be continually repeated, forming an *infinite* or *never ending series* of 3's.

A compound repetend is denoted by a point over the *first* and *last* repeating *figure:* thus, .2̇3̇4̇ signifies that 234 is to be continually repeated.

It may not be amiss, here to show how the *value* of any *repetend* may be found, or, in other words, to show how it may be *reduced to its equivalent vulgar fraction.*

If we attempt to reduce $\frac{1}{9}$ to a *decimal*, we obtain a continual repetition of the figure 1; thus, .1111 &c., that is, the repetend .1̇. The value of the repetend .1̇, then, is $\frac{1}{9}$; the value .222 &c., the repetend .2̇, will evidently be *twice* as much, that is, $\frac{2}{9}$. In the same manner, .3̇ $= \frac{3}{9}$, .4̇ $= \frac{4}{9}$, .5̇ $= \frac{5}{9}$, and so on to .9̇ which $= \frac{9}{9} = 1$.

1. What is the value of .8̇? *Ans.* $\frac{8}{9}$.
2. What is the value of .6̇? *Ans.* $\frac{6}{9} = \frac{2}{3}$.
3. What is the value of .3̇? —of .7̇? —of .4̇? —of .5̇? —of .9̇? —of .1̇?

If $\frac{1}{99}$ be reduced to a decimal, it produces .010101 &c., or the repetend .0̇1̇. The repetend .0̇2̇, being 2 times as much, must be $\frac{2}{99}$, and .0̇3̇, $= \frac{3}{99}$; and .4̇8̇, being 48 times as much, must be $\frac{48}{99}$, and .7̇4̇ $= \frac{74}{99}$; and so on.

If $\frac{1}{999}$ be reduced to a decimal, it produces .00̇1̇; consequently .00̇2̇ $= \frac{2}{999}$, and .03̇7̇ $= \frac{37}{999}$, and .4̇2̇5̇ $= \frac{425}{999}$, &c.

Hence, *to reduce* CIRCULATING DECIMALS *to* VULGAR FRACTIONS, we have this RULE. Make the GIVEN REPETEND the *numerator*, and the *denominator* will be as many 9's as there are REPEATING FIGURES.

4. What is the vulgar fraction equal to .7̇0̇4̇? *Ans.* $\frac{704}{999}$.
5. What is the value of .00̇3̇? —— .01̇4̇? —— .3̇2̇4̇? —— .01̇02̇1̇? —— .2̇4̇6̇3̇? —— .00̇2̇10̇3̇? *Ans. to last,* $\frac{701}{333333}$.

What is the value of .43̇?

In this fraction, the repetend begins in the second place, or place of *hundredths.* The first figure 4, is $\frac{4}{10}$, and the *repetend* 3, is $\frac{3}{90}$ of $\frac{1}{10} = \frac{3}{90}$. These two parts added together, give $\frac{4}{10} + \frac{3}{90} = \frac{36}{90} = \frac{13}{30}$, *Ans.* Hence

To find the value of a MIXED REPETEND,

RULE. Find the value of the TWO PARTS, *separately*, and add them together.

1. What is the value of .15̇3̇? $\frac{15}{100} + \frac{3}{990} = \frac{134}{990} = \frac{23}{150}$, *Ans.*
2. What is the value of .004̇7̇?
3. What is the value of .13̇8̇? —— of .16̇? —— of .41̇23̇? —— of .14523̇? —— of .345006̇? —— of .7161304̇?

It is plain, that circulates may be added, subtracted, multiplied, and divided, by first reducing them to their equivalent *vulgar fractions.*

6*d.* = $\frac{6}{12}$ of a shilling =. .5*s.*; which annexed to the 7*s.*, gives 7.5*s.* 7.5*s.* = $\frac{7.5}{20}$ of a pound =.375£. *Ans.*

The process may be written thus:

12	6.	*d.*
20	7.5	*s.*
	£. 375	*Ans.*

Hence we deduce the following

RULE I. Write the denominations under each other, beginning with the lowest, and the number which it takes of each to make one of the next higher, at the left hand with a perpendicular line between.

II. Begin with the first, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used,—the last quotient will be the decimal sought.

EXAMPLES.

2. Reduce 13*s.* 9½*d.* to the decimal of a pound.

4	3.
12	9.75
20	13.8125
	.690625£ *Ans.*

3. Reduce 10*oz.* 6*pwt.* 5*grs.* to the decimal of a pound.

4. Reduce 2*qrs.* 2*na.* to the decimal of a yard.

5. Reduce 7*cwt.* 3*qrs.* 17*lb.* 10*oz.* 12*drs.* to the decimal of a ton.

7*s.* and .5 of a shilling. .5 of a *s.* = $\frac{5}{10}$ of 12*d.* = $\frac{5}{10}$ of $\frac{12}{1}$ = $\frac{60}{10}$ = 6*d.* Therefore *Ans.* 7*s.* 6*d.*

The operation may be written thus: £ .375
20

s. 7.500
12

d. 6.000 *Ans.* 7*s.* 6*d.*

Wherefore we have this

RULE I. Multiply the given decimal by the number which it takes of the NEXT LOWER denomination to make ONE of that, and point off from the product as many figures for decimals as there are figures in the given decimal.

II. Multiply the DECIMAL PART of this product and point it off as before, and so on, and the several numbers at the left hand of the decimal points will constitute the value required.

EXAMPLES.

2. Reduce £ .690625 to its value.

£ .690625
20

s. 13.812500
12

d. 9.750000
4

qrs. 3.000000 *Ans.* 13*s.* 9*d.* 3*qrs.*

3. Reduce .3593 of a pound Troy to its value.

4. What is the value of .625 of a yard?

5. What is the value of .39538 + of a ton?

THEORETICAL QUESTIONS.

What have we seen in simple numbers? What, on the contrary, is therefore evidently true? How may we extend this reasoning? In pursuance of this order, what shall we have for the first place to the right hand of units? —— for the second? —— for the third? What are decimal fractions? What is the denominator to a decimal fraction? What do you infer from this? What is the denominator to the decimal .7854? How may a decimal be expressed in the form of a *vulgar fraction*? What are whole numbers and decimals called? How may we

consider *units*, with regard to INTEGERS AND DECIMALS? How is the value of every figure determined? What follows? What, on the contrary? How may decimals of *different denominators* be reduced to a *common denominator*? How are decimals read? What follows? What is Federal Money? How may denominations of this money be read? Give an example. What do expressions in dollars, cents and mills form? How do you add or subtract decimals? What is the rule for the multiplication of decimals? What, for their division? How do you reduce vulgar fractions to decimals? —— a compound number to the decimal of a higher denomination? —— the decimal of a higher denomination, to its value?

REDUCTION OF CURRENCIES.

156. Before the act of Congress in 1786, all calculations in money, throughout the United States, were made in *pounds, shillings, pence,* and *farthings*. But these denominations, although *the* SAME *in* NAME, were different in VALUE in different countries; *the* DOLLAR being reckoned in

ENGLAND,	at	4*s.* 6*d.*,	called ENGLISH OR STERLING money.
Canada & Nova Scotia,	at	5*s.*;	called CANADA currency.
The New-England States, Virginia, Kentucky, and Tennessee,	at	6*s.*,	called NEW-ENGLAND currency.
New-York, Ohio, & N. Carolina,	at	8*s.*,	called NEW-YORK currency.
Pennsylvania, N Jersey, Delaware, Maryland,	at	7*s.* 6*d.*,	called PENNSYLVANIA currency.
S. Carolina & Georgia,	at	4*s.* 8*d.*,	called GEORGIA currency.

EX. Reduce £7 13*s.* 4½*d.* to federal money.

157. To reduce ANY SUM, in either of the above named currencies to *Federal Money*,—RULE—*First reduce the* SHILLINGS, (PENCE *and* FARTHINGS, *if any,*) *to the decimal of a* POUND.—(See Art. 154.)

£7 13*s.* 4½*d.* = £7.6698 *nearly.*

STERLING MONEY.—*In all currencies* £1 = 20*s.* = 240*d.* 1 dollar, sterling money, is equal to 4*s.* 6*d.* = 54*d.* = $\frac{54}{240}$ = $\frac{9}{40}$ of a pound. As many times, therefore, as $\frac{9}{40}$ can be had in the given sum, so many DOLLARS there will be. Hence,—*To reduce sterling to federal money,*—RULE—DIVIDE THE GIVEN SUM BY $\frac{9}{40}$.

Supposing the above sum to be sterling money, we have

£ 7.6698 sterling money.
40

306.7920

To multiply by a fraction, *we multiply by the* DENOMINATOR, *and divide by the* NUMERATOR.—See Art. 80.

$ 34.088 federal money. *Ans.*

CANADA CURRENCY.—The *dollar* this currency is 5*s.* = 60*d.* = $\frac{60}{240}$ = ¼ of 1 pound. Therefore,—*To reduce Canada currency to Federal money,*—RULE—DIVIDE THE GIVEN SUM BY ¼, OR MULTIPLY IT BY 4.

ing the same sum as before to be Canada currency, we have

£7.6698 Canada currency.
4

$30.6792 Federal money, *Ans.*

NEW-ENGLAND CURRENCY.—1 dollar, in this currency, is 6*s.* = 72*d.* = $\frac{72}{240}$ = $\frac{3}{10}$ or .3 of a pound. Therefore,—*To reduce New-England currency to Federal money,*—RULE—DIVIDE THE GIVEN SUM BY .3.

.3) £ 7.6698 New-England currency.

$ 25.566 Federal money, *Ans.*

NEW-YORK CURRENCY.—The dollar is 8*s.* = 96*d.* = $\frac{96}{240}$ = $\frac{4}{10}$ or .4 of a pound. Therefore,—*To reduce New-York currency to Federal money,*—RULE—DIVIDE THE GIVEN SUM BY .4.

.4) £ 7.6698 New-York currency.

$ 19.19$\frac{2}{4}$ Federal money, *Ans.*

PENNSYLVANIA CURRENCY.—The dollar is 7*s.* 6*d.* = 90*d.* = $\frac{90}{240}$ = $\frac{3}{8}$ of a pound. Therefore,—*To reduce Pennsylvania currency to Federal money,*—RULE—DIVIDE THE GIVEN SUM BY $\frac{3}{8}$.

£ 7.6698 Pennsylvania currency.
8

3) 61.3584

$20.4528 Federal money, *Ans.*

GEORGIA CURRENCY.—The dollar is 4*s.* 8*d.* = 56*d.* = $\frac{56}{240}$ = $\frac{7}{30}$ of a pound. Therefore,—*To reduce Georgia currency to Federal money,*—RULE—DIVIDE THE GIVEN SUM BY $\frac{7}{30}$.

£ 7.6698 Georgia currency.
30

7) 230.0940

32.87\frac{1}{7}$ Federal money, *Ans.*

158. Hence, *To reduce Sterling money, and the currencies of Canada and the several United States, to Federal money;* we have this GENERAL RULE.—FIRST, reduce the *shillings*, &c. (if any) in the GIVEN SUM, to the decimal of a POUND; and then divide it by such fractional part, as 1 dollar is of 1 pound, in the given currency.

EXAMPLES.

1. Reduce £225 17*s.* 6*d.*, in each of the before named currencies, to Federal money. £225 17*s.* 6*d.* = £225.875; therefore

Ans.
- £225.875 Sterling money = 1003.88\frac{8}{9}$
- £225.875 Canada currency = $ 903.50
- £225.875 New-England currency = $ 752.91$\frac{6}{7}$
- £225.875 New-York currency = $ 564.68$\frac{3}{4}$
- £225.875 Pennsylvania currency = $ 602.33$\frac{1}{3}$
- £225.875 Georgia currency = $ 963.03$\frac{3}{7}$

2. Reduce £39 15*s.*, in the several currencies, to Federal money.
3. Reduce £17 to the currency of each of the before named countries.
4. Reduce £319 11*s.* 6½*d.*, in the several currencies, to Federal money.
5. Reduce 1*s.*, in the currency of each country, to Federal money.
6. Reduce £999 19*s.* 11½*d.*, in the several currencies, to Federal money.

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159. *To reduce Federal money to any of the before named currencies,* reverse the process in the foregoing operations, that is,

MULTIPLY the GIVEN SUM by such FRACTIONAL PART as 1 dollar is of 1 pound, and the product will be POUNDS *and decimals of a pound*, which reduce to *shillings, &c.*

EXAMPLES.

1. Reduce $514.25 to the several currencies.

Ans.
- £115.68375 = £115 13*s.* 8 *d.* Sterling money.
- £128.5625 = £128 11*s.* 3 *d.* Canada currency.
- £154.275 = £154 5*s.* 6 *d.* New-England currency.
- £205.7 = £205 14*s.* 0 *d.* New-York currency.
- £192.84375 = £192 16*s.* 10½*d.* Pennsylvania currency.
- £119.99167 = £119 18*s.* 4 *d.* Georgia currency.

2. Reduce $715 to the several currencies.
3. Reduce $999.999 to the several currencies.

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160. It is sometimes required, to reduce one currency to the PAR, or equality of another.

Let it be required, to reduce £500 Sterling money to New-York currency.

$1 is 4*s.* 6*d.* Sterling money = 54*d.* $1 is 8*s.* New-York currency = 96*d.*; that is, the value of any number of *pounds, &c.* Sterling money, is $\frac{96}{54} = \frac{16}{9}$ of the same in New-York currency; hence,

To reduce Sterling money to New-York currency,

RULE. MULTIPLY BY $\frac{16}{9}$. £500 Sterling money

16

9)8000

Is £888 17*s.* 9½*d.* New-York currency.

On the same principles, the learner can easily form for himself MULTIPLIERS, *by which to reduce* ANY *currency to the* PAR *of another.*

TABLE OF FOREIGN COINS.

Livres tournois of France	$ .18½	Johannes	$16.00
Franc do.	$ .18¾	Doubloon	$14.00
Silver Rouble of Russia	$ .75	Moidore	$ 6.00
Florin or Guilder of the U. N.	$ .40	English Guinea	$ 4.667
Mark Banco of Hamburgh	$ .33½	French Guinea	$ 4.36
Real of Plate of Spain	$ .10	Spanish Pistole	$ 3.773
Real of Vellon of do.	$ .05	French Pistole	$ 3.667
Milrea of Portugal	$1.24	English or French Crown, Sil.	$ 1.10
Tale of China	$1.48	Spanish Dollar	$ 1.00
Pagoda of India	$1.84	English shilling	$ .222
upee of Bengal	$ .50	Pistareen	$ .20

RATIO,

OR

THE RELATION OF NUMBERS.

1. What part of 1 penny is 3 farthings? *Ans.* $\frac{3}{4}$.
2. What part of 3 farthings is 1 penny? *Ans.* $\frac{4}{3}$; that is 1 penny (= 4 *far.*) is 1 time 3 farthings, and $\frac{1}{3}$ of another time.

161. In finding what PART one number is of another, we obtain what is called their RATIO, or the RELATION BETWEEN THEM.

3. What part of 5 quarts is 12 quarts? *Ans.* $\frac{12}{5}$; that is, $\frac{12}{5}$ is the ratio of 5 quarts to 12 quarts.
4. What is the ratio of £5 to £3? *Ans.* $\frac{3}{5}$.

162. The *first term* of a ratio is called the ANTECEDENT, and the *last* the CONSEQUENT.

163. The RATIO is expressed by making the *consequent* the NUMERATOR, and the *antecedent* the DENOMINATOR to a fraction.

5. What is the ratio of 8 yards to 13 yards? *Ans.* $\frac{13}{8} = 1\frac{5}{8}$.
6. What is the ratio of $12 to $3? *Ans.* $\frac{3}{12} = \frac{1}{4}$.
7. What is the ratio of £1 to 3*s.* 4*d*?

£1 = 240*d.*, and 3*s.* 4*d.* = 40*d.*; therefore, *Ans.* $\frac{40}{240} = \frac{1}{6}$.

164. RATIO may be defined *the number of times* ONE *number is contained in* ANOTHER; or the number of times *one quantity* is contained in *another quantity* of the SAME KIND.

8. What is the ratio of 9 to 18? *Ans.* $\frac{18}{9} = 2$.
9. What is the ratio of 13*s.* 4*d.* to £2 17*s.*? *Ans.* $\frac{684}{1160} = \frac{171}{290}$
10. What is the ratio of 5 to 3? —— of 3 to 5? —— of 15 to 40? —— of 40 to 15? —— of 17 to 28? —— of 28 to 17?
11. What is the ratio of 10 miles to 15 miles? *Ans.* $\frac{15}{10} = \frac{3}{2}$.
12. What is the ratio of 4 hours to 6 hours? *Ans.* $\frac{6}{4} = \frac{3}{2}$.

165. When TWO RATIOS are equal, the equality is called a *proportion*.

	hours	hours	miles	miles
Thus these four quantities	4	6	10	15

being such that the ratio between the *two first*, is equal to the ratio between the *two last*, are said to be PROPORTIONAL.

PROPORTION,

OR

THE RULE OF THREE.

It follows from the foregoing, that

166. PROPORTION IS AN EQUALITY OF RATIOS.

RATIO *exists between* TWO *quantities; but proportion* requires AT LEAST THREE. There may be a proportion between three quantities when *one of them is* REPEATED.

Thus, 3 *men* to 6 *men* as 6 *men* to 12 *men*. But it must be remembered, that proportion always requires FOUR TERMS.

167. That there is a proportion between the numbers, 4 6 10 15, is expressed by writing them thus:

4 : 6 : : 10 : 15.

This is read, the ratio of 4 to 6 equals the ratio of 10 to 15; or more concisely, 4 is to 6 as 10 to 15.

168. The FIRST and LAST terms of a proportion are called the EXTREMES, and the *two middle terms* the MEANS.

169. In a proportion there are TWO RATIOS, and consequently *two antecedents* and *two consequents*.

In the proportion 4 : 6 : : 10 : 15, the antecedents are 4 & 10, and consequents 6 & 15. The first ratio is $\frac{6}{4}$ (= $\frac{3}{2}$), and the second $\frac{15}{10}$ (= $\frac{3}{2}$). But these two ratios are *equal*, because the fractions which express them are equal. The two fractions $\frac{6}{4}$ & $\frac{15}{10}$ being equal, it follows, that if they be reduced to a common denominator, *the* NEW NUMERATORS *will be equal*; and, consequently, that 6 × 10, will be equal to 4 × 15. This is actually true; for 6 × 10 = 60, and 4 × 15 = 60. Hence,

If four numbers be proportional,

170. The product of the TWO EXTREMES will be EQUAL to the product of the TWO MEANS.

Consequently, *the product of the means divided by* EITHER *extreme will give the* OTHER *extreme.*

If 9 barrels of flour cost $63, how many barrels may be bought for $238?

Here the unknown quantity is the number of barrels bought for $238, which ought to have the same ratio to 9 barrels as $238 has to $63; whence we have

dollars		dollars		barrels		barrels
63	:	238	: :	9	:	.....

9

63)2142(34 barrels, *Ans.*
189

252
252

It must be recollected that *ratio exists between two quantities of the* SAME KIND, and between no others. Consequently, the first and second terms of a proportion must always be ALIKE.

171. In many cases, however, the things whose ratio is compared INCREASE, the *one* in the SAME RATIO as the *other* DIMINISHES.

As, for instance, the more men are about a piece of work, the less time it will take to do it; the quicker a man walks in proportion to another man, the less time he will require to go through a certain space; and so on. That is to say:

The RATIOS (of these things or their results) are INVERTED.*

Therefore, in all such cases, the ratio of the *two given terms of the same kind*,

* 'The RULE OF THREE has sometimes been distinguished into *direct* and *inverse*: The *Rule of Three Direct* is when *more* requires *more*, or *less* requires *less*, as in this example: If 3 men dig a trench 48 feet long in a certain time, how many feet will 12 men dig in the same time? Here it is obvious, that the *more* men are employed, the *more* work will be done; and consequently, in this case, MORE requires MORE. Again:—If 6 men dig 48 feet in a given time, how much will 3 men in the same time? Here *less* requires less, for the less men are employed the *less* work will be done.

is also to be inverted in the statement of the proportion, and then the operation of the rule of three is to be performed in the same manner as above.

If 4 men perform a certain piece of work in 12 days, how long will it take 8 men to perform the same work?

Evidently the *more* men are employed, the less time it will take to do the work, consequently the ratio of the time is the inverse of that of the men, and as this latter ratio is given, we write it inversely; that is, instead of writing 4 men : to 8 men, we write 8 men : to 4 men.

	men		men		days		days
Thus,	8	:	4	: :	12	:	.....

Multiplying the means, that is, the 2d and 3d terms together, and dividing the product by the known extreme, we obtain for the other extreme 6 days, *Ans.*

From the examples and illustrations now given, we deduce the following general

172. RULE I. Make that the THIRD TERM which is of the same NAME OR KIND with the ANSWER.

II. Consider from the nature of the question whether the answer should be GREATER or LESS than this third term—if *greater*, place the greater of the two remaining terms in the SECOND PLACE, and the other in the FIRST; and *vice versa*.

III. Multiply the SECOND and THIRD terms together, divide the product by the FIRST, and it will give the ANSWER.

NOTE. The first and second terms must be reduced to the lowest denomination mentioned in EITHER, and the third term to the lowest denomination mentioned; and then the answer will be in the same denomination that the third term was left in.

EXAMPLES.

1. If a horse travel 30 miles in 6 hours, how far will he travel in 1 hours at that rate? *Ans.* 55 *miles*

2. If the third of six be three
What will the fourth of twenty be?

3. There is a meadow, which 6 men usually mowed in 17 days; but the season being precarious, it is required to have it mowed in 3 days; how many men must be employed? *Ans.* 34.

4. If 10*lbs.* of tea cost \$8.75, what will 85*lbs.* cost? *Ans.* \$72.37½

5. If \$8.75 buy 10*lbs.*, how much will \$72.37½ buy? *Ans.* 85*lbs.*

6. If 85*lbs.* of tea cost \$72.37½, how much can be bought for \$8.75? *Ans.* 10*lbs.*

7. If a family of 7 persons consume 3 bushels of meal in one month, how many bushels will serve a family of 21 persons? *Ans.* 9.

172 *a.* Questions in this, and most other rules, may be performed by ANALYSIS; that is, upon general principles.

If 7 persons consume 3 bushels, 1 person, in the same time, would consume $\frac{1}{7}$ of 3 bushels, that is $\frac{3}{7}$ of 1 bushel; and 21 persons would consume 21 times as much, that is $21 \times \frac{3}{7} = \frac{63}{7} = 9$ bushels, as before.

8. Abraham had fine pears, 9 of which were worth 6 cents; and Billy had apples, 8 of which were worth 2 cents; required the number of apples that Billy must give Abraham for 15 of his pears. *Ans.* 40.

The *Rule of Three Inverse* is when *more* requires *less*, or *less* requires *more*, as in this example: If 6 men dig a certain quantity of trench in 14 hours, how many hours will it require 12 men to dig the same? Here *more* requires *less*; that is, 12 men being more than 6 will require *less* time. Again:—If 6 men perform a piece of work in 7 days, how long will 3 men be in performing the same? Here *less* requires *more*; for the number of men BEING LESS, will require MORE time.

ANALYSIS. Abraham's pears are evidently worth $\frac{2}{3} = \frac{8}{12}$ of a cent apiece; hence 15 are worth $15 \times \frac{8}{12} = 10$ cents. Billy's apples are worth $\frac{2}{5} = \frac{4}{10}$ of a cent apiece; consequently 10 cents (the worth of 15 pears) would buy $(10 \div \frac{4}{10} =)$ 40 apples, as before.

9. If 60 gallons of water in one hour's time, fall into a cistern that will hold 200 gallons, and by a pipe in the same cistern there runs out 45 gallons in one hour, in what time will the cistern be filled? *Ans.* 13*h.* 20*m.*

10. If 40*lbs.* of tobacco cost $6.80, what will 100*lbs.* cost? *Ans.* $17.

By analysis. If 40*lbs.* cost $6.80, 1*lb.* would cost $\frac{6.80}{40} = 17$ cents, and 100*lbs.* would cost 100 times as much, and 100 times 17 *cents* would be $17, as before.

NOTE. It will be very good exercise for the scholar to prove his questions by analysis.

11. If 40 yards of cotten cloth cost $4, what cost 7000 yards? *Ans.* $700.

12. When eggs are 12½ cents a dozen, how much will 117 come to?

13. If a field be sufficient for 6 cows 90 days, how long will it serve 20 cows? *Ans.* 27 *days.*

14. Charles told Frederick that his purse and money were worth $6.12½, and that the money was worth six times as much as the purse; pray, how much was there in it? *Ans.* $5.25.

15. Just sixteen yards of German serge,
For ninety dimes had I;
How many yards of that same cloth
Will fourteen Eagles buy? *Ans.* 248*yds.* 3*qrs.* 2$\frac{2}{5}$*na.*

16. If the moon moves through 13° 10′ 35″ in one day, in what time does it perform one revolution? *Ans.* 27*d.* 7*h.* 43*m.*

17. Some sportsmen having placed a fox 100 yards distant from two hounds, and starting both together, the hounds ran 2½ times faster than the fox; how far will the fox run before the hounds overtake her? *Ans.* 66$\frac{2}{3}$*yds.*

18. A certain building was erected by 12 men in 8 months; but the same being demolished, it is required to rebuild it in 3 months; how many men must be employed? *Ans.* 32.

19. If 3 men or 4 women, will do a job of work in 56 days, how long will it take one man and one woman to do the same? *Ans.* 96 *days.*

20. If a man perform a journey in 15 days, when the days are 12 hours long, what time will it take when the days are 10 hours long? *Ans.* 18.

21. How much persian, that is ¾ yard wide will it take to line 25$\frac{1}{2}$*yds.* 5 quarters wide? *Ans.* 42$\frac{1}{2}$*yds.*

22. As I was beating on the forest ground,
Up starts a hare before my old grey-hound;
The dog, being light of foot, did fairly run
Unto her fifteen rods, just twenty-one.
The distance that she started up before,
Was fourscore sixteen rods, just and no more;
Now this I'd have you unto me declare,
How far he ran before he caught the hare? *Ans.* 336 *rods.*

COMPOUND PROPORTION,

OR

DOUBLE RULE OF THREE.

173. When the ratio of the quantity required to the given *quantity* of the same kind depends upon several circumstances,

it is called **COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.**

If 12 men, in 15 days of 12 hours each, dig a ditch 30 feet long, 6 wide, and 3 deep; in how many days will 60 men dig a ditch 300 feet long, 8 wide, and 6 deep, when the days are but 8 hours long?

Here are *five circumstances*, upon which the number of days required depends; 1st the number of men employed; 2ndly the hours in a day; 3dly the lengths; 4thly the widths; and 5thly the depths of the ditches. We will consider the question in relation to each of these circumstances; and

1st. *The number of men employed.* Had all the circumstances in both cases been alike, except the number of men and the number of days, the question would have consisted only in finding how many days it would take 60 men to perform the work which 12 men had done in 15 days; in which case we should have

60 men : 12 men :: 15 days : ..... days.

2nd. *Hours in a day.* But the first labourers worked 12 hours in a day, and the others only 8; *less* hours will require *more* days, whence we have

8 hours : 12 hours :: 15 days : ..... days.

3d. *Length of the ditches.*

30 feet : 300 feet :: 15 days : ..... days.

4th. *Width of do.*

6 feet : 8 feet :: 15 days : ..... days.

5th. *Depth of do.*

3 feet : 6 feet :: 15 days : ..... days.

But in reality, the ratio of the 15 days to the number of days required, depends upon the ratios of all these other circumstances *combined together*. Hence, after stating the proportion, which is done in this way,

Men	60	:	12	common term
Hours	8	:	12	:: 15 days : ..... days,
Length	30	:	300	
Width	6	:	8	
Depth	3	:	6	

We divide the continued product of all the second terms *multiplied by the* THIRD, by the continued product of all the first terms: and it gives the fourth term, or *Answer*, (= 120 *days*.)

But the ratio of the first and second terms respectively, are $\frac{12}{60}$, $\frac{12}{8}$, $\frac{300}{30}$, $\frac{8}{6}$, & $\frac{6}{3}$; and the ratio of the number of days given to the number of days required, is equal to the product of all these ratios; that is, $= \frac{12 \times 12 \times 300 \times 8 \times 6}{60 \times 8 \times 30 \times 6 \times 3} = \frac{2073600}{259200} = 8$.

Hence, if we look at the process, and at the same time turn to Art. 122, *a & b*, we shall evidently see that the operation may be very much abridged. For it is obvious, that we may cancel and divide, so as to reduce $\frac{12 \times 12 \times 300 \times 8 \times 6}{60 \times 8 \times 30 \times 6 \times 3}$ to $\frac{4 \times 2 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 1 \times 1}$ $= \frac{4 \times 2}{1} = 8$. Hence it appears, that the ratio of the given number of days 15, to the number of days required, is 8; that is, that the number of days required is 8 times 15. This is actually true, for 8 times 15 = 120 days, *Answer*, as before.

Now the simplest turn of thought, will lead to the conclusion, that the above *cancelling &c.*, may be performed upon the numbers as they stand in the proportion; for the first and second terms are each to be regarded as the terms of a fraction, expressing the ratio. By this application, we have for the results

60	1	:	12	4
8	1	:	12	2
30	1	:	300	10
6	1	:	8	1
3	1	:	6	1

Common term.
:: 15 days
Then 8
—
120 days, *Ans.*, the same as before.

From this example and illustration we deduce the following general

174. RULE I. Make that the third term which is of the same name or kind with the answer.

II. Of the REMAINING NUMBERS, take *any two* that are of the SAME KIND, and place them according to the direction given in simple proportion: and then two *others* of the same kind, and so on, till all are used.

III. Multiply the third term by the continued product of the second terms, and divide the result by the continued product of the first terms, and the quotient will be the fourth term, or answer. Or,

Before multiplying *cancel and divide*, as shown above.

EXAMPLES.

1. If a man travel 494 miles in 13 days, when the days are 10 hours long; how far will he travel in 15 days, when the days are 12 hours long? *Ans.* 684 *miles.*

2. If a man perform a journey of 494 miles in 13 days, when the days are 10 hours long; how long will it take him to perform a journey of 684 miles, when the days are 12 hours long? *Ans.* 15 *days.*

3. If a journey of 684 miles can be peformed in 15 days of 12 hours each, how long must the days be, to perform a journey of 494 miles in 13 days? *Ans.* 10 *hours.*

4. If 18 men build a wall 40 feet long, 3 thick, and 16 high in 12 days; how many men will build a wall 360 feet long, 8 thick, and 10 high in 60 days? *Ans.* 54.

5. If 40 acres of grass be cut down by 8 men in 7 days, how many acres will 24 men cut in 28 days? *Ans.* 480 *acres.*

6. If 14 horses eat 56 bushels of grain in 16 days, how many bushels will 20 horses eat in 24 days? *Ans.* 120.

7. If 18 rods of ditching be done by 3 men in 6 days, how many rods will be done by 8 men in 4 days? *Ans.* 32.

8. If 18 men eat 13*s.* worth of bread in 6 days, when wheat is sold at 12*s.* per bushel, in how many days will 30 men eat 43½*s.* worth, when wheat is 10*s.* a bushel? *Ans* 12.

9. If 48 bushels of corn yield 576 bushels in one year, how much will 240 bushels yield in 6 years? *Ans* 17280.

THEORETICAL QUESTIONS.

In finding what PART one number is of another, what do we obtain? What is the first term of a ratio called? —— the last? How is the ratio expressed? How may it be defined? What is proportion? To denote that there is a proportion between numbers, how do you write them? What if four numbers be proportional? What follows? What if the things whose ratio is compared INCREASE, the *one* in the same ratio as the other *diminishes*? What is the first part of the RULE? —— the second? —— the third? —— the Note? What is compound proportion or double rule of three? What the RULE?

EXAMPLES FOR PRACTICE.

1. If 14 men in 15 days build 16 rods of wall,
How many men must added be, to do it in 2 that's all? *Ans.* 91.

2. A merchant bought 795 yards of cloth for $107.50; he has still $427.50 which he wishes to lay out in the same cloth, at the former price yards will it purchase? *Ans.* 3161*yds.* $\frac{2}{3}$.

3. There is a cistern having 3 pipes which will empty it in 6*h.* 45*m.*; how many pipes of the same capacity will empty it in 15 minutes?
Ans. 9.

4. When wheat is sold at $ .93, the penny loaf weighs 12*oz.*; what must it weigh when wheat is sold at $1.24? *Ans.* 6*oz.*

5. If from a rod of 6 feet long, a shadow 10 is made,
I ask the steeple's height in yards, that's 90 feet in shade? *Ans* 18.

6. A merchant bought a bale of cloth containing 240*yds.*, at the rate of $7½ for 5*yds.*, and sold it at the rate of $11½ for 7*yds.*; did he gain or lose, and how much? *Ans. He gained* $25.714+.

7. Bought threescore barrels for 3 times as many pounds, and sold them again for 4 times as much; if they had cost me as much as I sold them for, how should I have sold them, to gain at the same rate?
Ans. £320.

8. A factor bought a certain quantity of broadcloth and drugget, which together cost £81; the quantity of broadcloth was 50*yds.* at 18*s.* a yard, and for every 5 yards of broadcloth he had 9*yds.* of drugget; how many yards of drugget had he? and what did it cost him per yard?
Ans. 90*yds. at* 8*s.*

9. If ¼ of a yard of cloth cost ⅛ of a shilling, what will ½ of a yard cost?
Ans. 2*s.* 4*d.*

10. If the penny white-loaf weigh 7*oz.* when a bushel of wheat costs 5*s.* 6*d.*, what is the bushel worth when the penny white-loaf weighs 2½*oz.*?
Ans. 15*s.* 4½*d.*

11. A person failing in trade, was owing to his creditors $11573, of which he could pay only $ .49 on the dollar; how much would that be?
Ans. $5670.77

12. Whereas an eagle and a gent just three score yards did buy,
How many yards of that same cloth for fifteen dimes had I?
Ans. 8*yds.* 3*qrs.* 3*na.*+

13. If 7 men can reap 84 acres in 12 days, how many men can reap 100 acres in 5 days? *Ans.* 20.

14. If ⅝*yd.* cost $½, what will 10½*yds.* cost? *Ans.* $24½

15. If 1½*lb.* of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{3}{4}\frac{2}{5}$ of a pound cost?
Ans. 4*d.* $3\frac{42}{100}\frac{71}{25}$*qrs.*

16. Suppose 650 men in a garrison, have provisions for two months; how many men must leave the garrison, that the same provisions may last 3 months longer? *Ans.* 390.

17. If 6 men build a wall 20 feet long, 6 high, and 4 thick in 9 days, in what time will 24 men build one 200 feet long, 8 high, and 6 thick?
Ans. 45 *days.*

18. A merchant failing, owes in all £1490 5*s.* 10*d.*, and has in money, goods, and recoverable debts, £784 17*s.* 4*d.*; if these things be delivered to his creditors what will they get on the pound? *Ans.* 10*s.* 6*d.* 1*qr.* $\frac{20998}{31767}$.

19. If 3 masters, who have each 8 apprentices, in 5 weeks, each week 6 days, earn $360; how much will 5 masters, who have each 10 apprentices, earn in 8 weeks, each week 5½ days, their daily wages being the same?
Ans. $1100.

20. A regiment of soldiers consisting of 850 men are to be clothed, each suit to contain 3¼ yds. of cloth, which is 1¾ yds. wide, and lined with shalloon ¾ yard wide; how many yards of shalloon will it take?
Ans. 6941*yds.* 2*qrs.* 2¾ *na.*

21. If 80000 cwt. of ammunition were to be removed from a place in 9 days, and that in 6 day's time 4500 cwt. was carried away with 18 horses; how many horses will it require to carry away the remainder in 3 days?
Ans. 604.

22. If 30 men can perform a piece of work in 11 days, how many men will accomplish another piece 4 times as large in a fifth part of th

23. If a cistern of 17½ feet long, 10½ broad, and 13 deep, hold 546 barrels of water, how many barrels will fill a cistern that is 16 feet long, 7 broad, and 15 deep? *Ans.* 384.

24. Lent a friend $292 for 6 months; some time after, he lent me $806; how long may I keep it to balance the favour? *Ans.* 2*mo.* 6*d.*

25. How many pears can I have for $1.35, if 70 chesnuts are worth 50 almonds, and 48 almonds 1 pomegranate, and 28 lemons 18 pomegranates, and 25 pears 10 lemons, and 108 chesnuts cost 2½ cents? *Ans.* 375.

26. If 21 men could perform a work in 17 days, and 16 men be added to them after the second day, how much time will be saved by it?

Ans. $6\frac{11}{17}$ *days.*

27. If 12 shoemakers, in 2 weeks, make 36 pair of men's and 24 pair of women's shoes; how many pair of each sort, would 18 shoemakers make in 5 weeks? *Ans.* 135 *pair of men's, and* 90 *pair of women's.*

28. If A raises goods worth $30 to $40 and gives 9 months credit, how high must B raise goods worth $27 to give 10 months credit? *Ans.* $37.

29. A wall was to be built 700 yards long in 29 days; and after 12 men had worked 11 days, they had completed only 220 yds.; how many men must be added to finish it in the time proposed? *Ans.* 4.

30. A ship's crew of 300 men. are provided with provisions for 12 months, so that each man is allowed 30 oz. per day; but having been 6 months on their voyage, they find it will take 9 months more before they can finish it, and having lost 50 of their number, it is required to find the daily allowance of each man during the last 9 months. *Ans.* 24 *ounces.*

PRACTICE.

175. Practice is a contraction of the **RULE OF THREE**, when the first term happens to be a unit.

176. One number is said to be an **ALIQUOT PART** of another, when it exactly measures that other number.

Thus, 25 cents is an *aliquot part* of 1 dollar, because 25 cents exactly measures 1 dollar; that is, it is contained just 4 times in 1 dollar. 4 months is an *aliquot part* of 1 year, because it is contained just 3 times in 1 year, &c.

TABLE OF ALIQUOT PARTS.

Cts.	Parts of $1.	Mo.	Parts of 1 year.	Days	Parts of 1 mo.	Parts of £1.	Parts of 1 shilling.
50	$\frac{1}{2}$	6	$\frac{1}{2}$	15	$\frac{1}{2}$	10*s.* = $\frac{1}{2}$	6*d.* = $\frac{1}{2}$
$33\frac{1}{3}$	$\frac{1}{3}$	4	$\frac{1}{3}$	10	$\frac{1}{3}$	6*s.* 8*d.* = $\frac{1}{3}$	4*d.* = $\frac{1}{3}$
25	$\frac{1}{4}$	3	$\frac{1}{4}$	$7\frac{1}{2}$	$\frac{1}{4}$	5*s.* = $\frac{1}{4}$	3*d.* = $\frac{1}{4}$
20	$\frac{1}{5}$	2	$\frac{1}{6}$	6	$\frac{1}{5}$	4*s.* = $\frac{1}{5}$	2*d.* = $\frac{1}{6}$
$12\frac{1}{2}$	$\frac{1}{8}$	1	$\frac{1}{12}$;	5	$\frac{1}{6}$	3*s.* 4*d.* = $\frac{1}{6}$	$1\frac{1}{2}$ = $\frac{1}{8}$
$6\frac{1}{4}$	$\frac{1}{16}$		or $\frac{1}{8}$ of	3	$\frac{1}{10}$	2*s.* 6*d.* = $\frac{1}{8}$	1*d.* = $\frac{1}{12}$
5	$\frac{1}{20}$		3 mo.			1*s.* 8*d.* = $\frac{1}{12}$	

177. 1. What is the cost of 1576*yds.* at $ .33⅓ per yard?

Were the cloth $1 per yard, it is plain, it would cost just as many dollars as there are yards, viz. $1576; consequently, the cost of it at 33⅓ cents (= ⅓ of a dollar), will be ⅓ as much, that is, 3)1576 = $525.33⅓, *Ans.*

2. What is the cost of 1262 yards at $ .75 per yard?

cts.		OPERATION. dollars.
50	½	1262
		631 cost at 50 cents.
25	½ of 50cts.	315.50 " at 25 cents.
		$946.50 " at 75 cents, *Ans.*

At $1 per yard, it would be $1262;—at 50 cents it would be ½ as much, that is $631. But the price is 75 cents, that is, 25 cents more than 50. The cost at 25 cents, it is evident, would be ½ as much as at 50 cents, that is, $315.50. And it is plain, that the cost at 75 cents, will be the sum of these two, viz., the cost at 50 cents and at 25; that is $631 + $315.50 = $946.50, *Ans.*

3. What is the cost of 3746*yds.* at 4*d.* per yard?

The cost at 1*s.* per yard would be 3746*s.*; hence the cost at 4*d.* will be ⅓ as much. *Ans.* £62, 8*s.* 8*d.*

4. What is the cost of 4937*yds.* at 9*d.* per yard?

It is evident that the cost at 6*d.* and at 3*d.* added together will be the cost at 9*d.* *Ans.* £185, 2*s.* 9*d.*

5. What is the cost of 473*yds.* at 6*s.* 8*d.* per yard?

6*s.* 8*d.* = ⅓ of a pound. *Ans.* £157, 13*s.* 4*d.*

178. 6. What is the cost of 287*yds.* at 7*s.* 6*d.* per yard?

287*yds.* at £1 =		287£		
			s.	
10*s.*	½	143	10	
5*s.*	¼ of 10*s.*	71	15	
2*s.*6*d.*	¼ of 5*s.*	35	17	6
	Ans.	£251	2	6*d.*

The cost at 10*s.*, it is evident, will be ½ as much as the cost at £1; the cost at 5*s.* will be ½ as much as the cost at 10*s.*; and the cost at 2*s.* 6*d.* will be ¼ as much as the cost at 5*s.* And it is also evident, that the cost at 10*s.*, at 5*s.*, and at 2*s.* 6*d.* added together, will be the cost at (10*s.* + 5*s.* + 2*s.* 6*d.* =) 17*s.* 6*d.*

7. What is the cost of 146*yds.* at 14*s.* 9*d.* per yard? *Ans.* £107, 13*s.* 6*d.*

8. What cost 218½*yds.* at 12*s.* 6*d.* per yard? *Ans.* £136, 11*s.* 3*d.*

9. What cost 156*yds.* at £3, 6*s.* 8*d.* per yard? (The price at £3 would be £468.) *Ans.* £520.

10. What cost 999⅛ tons, at £19 19*s.* 11*d.* 3*qrs.* per ton? *Ans.* £19992, 6*s.* 5¾*d.*

THEORETICAL QUESTIONS. What is practice? When is one number said to be an aliquot part of another?

INTEREST.

179. Interest is an allowance made for the use of money.

It is computed at a certain RATE PER CENT.; that is at a certain number of dollars for each *hundred* dollars, or at a certain number of pounds for each *hundred* pounds, &c.*

The rate established by law in New-England is 6 per cent.; that is,

$6 are allowed for each $100, 6 cents for each 100 cents, £6 for each £100, and so on; in other words,

$\frac{6}{100}$ *of the sum* LENT *or* DUE *is allowed for the use of it* ONE YEAR.

In New-York the rate established by law is 7 per cent.; that is, in New-York $\frac{7}{100}$ *of the sum lent or due is allowed for the use of it one year.*

* And is understood to be PER ANNUM, that is, by the year.

SUPPOSE THE SUM LENT OR DUE TO BE $1.

In New-England the interest of it for 1 year, would be $\frac{6}{100}$ of a dollar; that is, 6 cents, which expressed decimally, would be . . . . . $.06

In New-York it would be 07

And so of any other *rate per cent.*

180. PER CENT. signifies *by the hundred*; rate per cent. is, therefore, a decimal carried to TWO PLACES. All rates *lower* than hundredths are parts of *one per cent.*

1 per cent. is 01
½ per cent. is half of 1 per cent., that is, ...005
¼ per cent. is a fourth of 1 per cent., that is, ...0025
¾ per cent. is 3 times ¼ per cent., that is, ...0075

What is the decimal expression of 2½ per cent.? —— 3 per cent.? —— 3½ per cent.? —— 4 per cent.? —— 4½ per cent.? —— 5 per cent.? —— 5½ per cent.? —— 5¼ per cent.? —— 8 per cent.? —— 8¼ per cent.? —— 9 per cent.? —— 9¾ per cent.? —— 10 per cent.? —— 10¼ per cent.? —— 10¾ per cent.? —— 12½ per cent.?

181. PRINCIPAL is the money AT INTEREST.

182. AMOUNT is the *principal and interest.*

1. What is the interest of $19 for 1 year at 6 per cent.?

The interest of $1 for 1 year is $ .06, that is 6 cents; consequently the interest of $19 will be 19 times as much, that is, 19 times .06; or, which is the same thing, .06 times 19. Thus,

$19	**183.** Hence to find the interest of any sum for 1 year, we have this RULE.—*Multiply it by the* RATE PER CENT. *written as a* DECIMAL FRACTION.
.06	
$1.14	

2 What is the interest of $72.15 for one year at 3½ per cent.?

$72.15	Here are five decimal places in the factors; therefore we point off five places for decimals in the product, and obtain for the answer $2, 52 cents, 5 mills, and $\frac{25}{100}$ of a mill. The *parts of a mill* are generally disregarded as of no material value.
.035	
36075	
21645	
$2.52525	

3. What is the interest of $35 for 1 year at 7 per cent? *Ans.* $2.45.

4. What is the interest of $42.05 for 1 year, at 4 per cent.? —— at 4½ per cent.? —— at 5 per cent.? —— at 5½ per cent.? —— at 7 per cent.? —— at 7½ per cent.? —— at 8½ per cent.? —— at 9 per cent.? —— at 10 per cent.? —— at 10½ per cent.? —— at 11¾ per cent.?

Ans. to the last, $4.94.

5. What is the interest of $715 for 1 year, at 7 per cent.? *Ans.* $50.05

184. It is plain that the interest of any sum for 2 years, will be *double* the interest for 1 year; that for 3 years it will be 3 times the interest for 1 year, &c.

Hence, *to find the interest of* ANY SUM *for* ANY NUMBER *of years,* we have this

Multiply the interest for ONE YEAR by the NUMBER OF

he interest of $100 at 6 per cent. for 1 year? —— for 2 r 3 years?

7. What is the interest of $85.62½ for 5 years, at 6 per cent.?

$85.625
.06

5.13750, interest for 1 year.
5, *number of years.*

Ans. $25.68750, interest for 5 years.

8. What is the interest of $125 for 3 years, at 7 per cent.? *Ans.* $26.25.
9. Interest of $17.32½ for 4½ years, at 6 per cent.? *Ans.* $4.81+.
10. ..... of $712.05 for 7 years, at 5 per cent.? *Ans.* $249.21¾.
11. ..... of $119.57½ for 5 years, at 4½ per cent.? *Ans.* $26.97+.
12. ..... of $312.05 for 3 years at 7 per cent.? *Ans.* $65.53+.

MONTHS AND DAYS.

185. The interest of $1 for 1 year, at 6 per cent., is
.06 cents; which is
.01 cent for 2 months,
.005 mills (or half a cent) for 1 month of 30 days; and
.001 mill for every 6 days.

From which it appears, that the interest of $1, at 6 per cent., will be *half as many* CENTS as there are *months*, and $\frac{1}{6}$ as many mills as there are days. Hence, if we take HALF THE MONTHS, AND $\frac{1}{6}$ OF THE DAYS, it will give the interest, in *cents and mills*, of $1 at 6 per cent., for ANY GIVEN TIME.

13. What is the interest of $1 at 6 per cent., for 5*mo.* 20*d.*?

Half the months is 2½; hence the interest for 5 months is 2½ cents, or $ .025. $\frac{1}{6}$ of the days is 3⅓; consequently the interest for 20 days is 3⅓ mills, and the interest for the whole time, $ .028⅓.

Or, we may add the odd month (= 30 days) to the 20, and take $\frac{1}{6}$ of the sum. Thus, we would have for 4 months 2 cents, for the odd month and days, (= 50*d.*,) 8⅓ mills; and the whole would be $ .028⅓, as before.

14. What is the interest of $1 at 6 per cent., for 7 months? —— for 6mo. & 15 days? —— for 8 months? —— 8mo. 27d.? —— for 9 months? —— for 10 months? —— for 10mo. 19d.? —— for 11 months? —— for 11mo. 21d. *Ans. to last,* $ .058⅓.

186. The interest of $2 is evidently twice as much as the interest of $1, the interest of $3 is 3 times as much, &c.

Hence, *to find the interest of* ANY SUM *for* ANY GIVEN TIME, *at 6 per cent.*, we have this

RULE. Multiply the GIVEN SUM by *half the even number* of months, written as HUNDREDTHS, and $\frac{1}{6}$ *of the number of days*, written as THOUSANDTHS.*

15. What is the interest of $217, 62½ at 6 per cent., for 14 years 9 months and 20 days?

Half the even number of months is 98, written as *hundredths* thus, .88; $\frac{1}{6}$ of the number of days (30+20) is 3⅓, written as *thousandths* thus, .008⅓; the whole making .888⅓, for the multiplier.

Hence $ 217.625
.888⅓

1 741 000
17 410 00
174 100 0
72 541+

Ans. $193.323 541+

* When the days are less than 6, it is evident, we must put 0 in the place of wri

16. What is the interest of $114 at 6 per cent. for 9mo. 14 days?
Ans. $5.396

17. Interest of $194 for 4mo. 12 days? *Ans.* $4.268.
18. ..... of $263.48 for 2mo. 21 days? *Ans.* $3.556.
19. ..... of $318 for 10mo. 16 days? *Ans.* $16.748.
20. ..... of $418 for 1Y. 7mo. 17 days? *Ans.* $40.894.
21. ..... of $268.44 for 3Y. 5mo. 26 days? *Ans.* $56.193.

187. 7 per cent is $\frac{1}{6}$ more than 6 per cent. Hence

To find the interest of any sum at 7 per cent.

Rule. First find the interest at 6 per cent.; then add to it $\frac{1}{6}$ part of itself.*

22. What is the interest of $165 for 2Y. 7mo. at 7 per cent.?

Half the even number of months (30), written as directed, is .15 & the odd month .005

Sum .155, the multiplier.

$165 principal.
.155 interest of $1 for the given time [at 6 per cent.
825
825
165

$25.575 interest at 6 per cent.
$+\frac{1}{6}=4.261$

Ans. $29.836 interest at 7 per cent.

23. What is the interest of $519.20 for 3Y. 9mo. at 7 per cent?
24. Interest of $145 for 2Y. 3mo.? *Ans.* $22.837.
25. ..... of $46 for 2Y. 4mo.? *Ans.* $7.513.
26. ..... of $712 for 3 weeks? *Ans.* $2.907.
27. ..... of $368.84 for 3Y. 6mo.? *Ans.* $90.365.
28. ..... of $384.25 for 1Y. 9mo. 15d.? *Ans.* $48.19.
29. ..... of $910.50 for 3Y. 9mo. 26d? *Ans.* $243.609.

188. Universally *when interest is at* any other *rate than* 6 per cent.,

Rule. Find the interest at six per cent.; then add to it, or subtract from it, such part as the rate required *exceeds* or *falls short* of 6 per cent.*

30. What is the interest of $45 for 10 months, at 4½ per cent.?

$45
.05
$\frac{1}{4}$)2.25
5625

$1.6875, *Ans.*

4½ per cent. is ¾ of 6 per cent.; therefore, from the interest at 6 per cent we subtract ¼ of itself, and obtain the interest at 4½ per cent.

31. Interest of $354 for 14 months at 5 per cent.? *Ans.* $20.65.
32. ..... of $501.21 for 5mo. 21d., at 4 per cent.? *Ans.* $9.523.
33. ..... of $721.09 for 11mo. 6d., at 8 per cent.? *Ans.* $53.841.
34. ..... of $999.99 for 1mo. 17d., at 10 per cent.? *Ans.* $13.055.
35. ..... of $95 for 21mo. 24d., at 12½ per cent.? *Ans.* $21.573.
36. ..... of $777.77 for 18d., at 15 per cent.? *Ans.* $5.833.
37. What is the amount of $52 for 27mo., at 6 per cent.? *Ans.* $59.02.
38. Amount of $101.72 for 9 mo. 12d., at 7 per cent.? *Ans.* $107.297.
39. ..... of $514.51 for 17mo. 18d., at 8 per cent.? *Ans.* $574.879.

* Or, find the interest for the even years, and take parts for the *months and days*.

40. Amount of $701.01 for 27mo. 19d., at 10 per cent.? *Ans.* $862.437.
41. ..... of $888.888 for 88mo. 8d., at 12½ per cent.? *Ans.* $1706.171.

189. To find the interest of any sum in *pounds, shillings, pence, &c.*

RULE. *Reduce the* SHILLINGS, &c. *to the decimal of a* POUND, *and proceed as in* FEDERAL MONEY.

42. Interest of £351 1s. for 9½mo., at 6 per cent.? *Ans.* £16 13*s.* 5½*d.*
43. ..... of £126 12s. for 16 weeks, at 6 per cent.? *Ans.* £2 7*s.* 3*d.*
44. ..... of £500 for 20 weeks, at 7 per cent.? *Ans.* £13 12*s.* 2½*d.*
45. ..... of £400 10s. for 18mo., at 6 per cent.? *Ans.* £36 0*s.* 9½*d.*
46. ..... of £378 3s. 8½d. for 9mo. 5d., at 6 per cent.? *Ans.* £17 6*s.* 8*d.*

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190. *The time, rate per cent., and amount given, to find the principal.*

1. What sum of money, put to interest at 7 per cent., will in 14 months, amount to $216.333⅓?

The amount of $1, for the given rate and time, is $1.816⅔; hence,

Am't of $1.		Given am't.	Princ'l of former.		Princ'l of latter.
$1.816⅔	:	$216.333⅓	: : $1	:	$200, *Ans.*

Whence we have this

RULE. *Divide the* GIVEN AMOUNT *by the* AMOUNT OF ONE DOLLAR, *for the given rate and time.*

2. What sum of money, put to interest will in 16 months, amount to $340.20? *Ans.* $315.
3. What principal, will, in 18 months, amount to $299.75, at 6 per cent.? *Ans.* $275.
4. What principal, at 6 per. cent., will, in 13mo. 9d., amount to $533.25? *Ans.* $500.

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191. *The prices at which goods are* SOLD, *and the gain or loss* PER CENT., *being given, to find the* PRIME COST.

5. If by selling tea at $ .09 a pound, there is gained 20 per cent., what was the prime cost?

The selling price of $1's worth, to gain 20 per cent., would be $1.20; hence, by the foregoing rule, $1.20).90($ .75, *Ans.*

6. If by selling cloth at $2.18¾ per yard, there is lost 12½ per cent., what was the purchase price?

The selling price of $1's worth, to lose 12½ per cent., would be $ .87½; whence by the rule, $ .875)2.1875($2.50, *Ans.*

7. If by selling calico at $ .16 per yard, there is lost 20 per cent., what was the first cost? *Ans* $ .20.
8. Sold a hogshead of sugar for $90, and by so doing gained 5 per cent.; how much did I give for it? *Ans.* $85.714+.
9. Sold chocolate at $ .22½ a pound, and thus lost 10 per cent.; what did it cost me a pound? *Ans* $ .25.
10. If by selling molasses at $ .50 per gallon, there is gained 50 per cent., what was the first cost? *Ans.* $ .33333⅓.
11. If 126 gallons of wine be sold at 15s. per gallon, whereby is gained 20 per cent., what was the prime cost? *Ans.* 12*s.* 6*d. per gal.*

DISCOUNT.

192. DISCOUNT is an allowance *for* the payment of money *before it becomes* DUE.

193. PRESENT WORTH is that sum, which, if put to interest, would, at the given rate and time, amount to the sum or debt to be discounted.

12. A owes B $544.84, payable in 1 year, without interest, but he wishes to pay it immediately; how much ought he to pay when the usual rate is 6 per cent.?

It is plain, he ought to pay such a sum, as put to interest, would, in 1 year, amount to $544.84. The question, therefore, does not differ from the foregoing; that is, it does not differ from finding the PRINCIPAL, *the amount, rate per cent, and time being given.* The *answer* is $514.

13. What is the present worth of $350, due 6 months hence, discounting at the rate of 6 per cent.? *Ans.* $339.805.

14. What is the present worth of $700, due 2 years and 9 months hence, discounting at 7 per cent.? *Ans.* $623.608+.

15. What ready money will discharge a debt of $1595, due 5 months 20 days hence, at 6 per cent? *Ans.* $1541.326.

16. Bought goods to the amount of $250, ready money, and sold them for $300, payable 9 months hence; whether did I gain or lose, and how much? *Ans.* $37.081 *gain.*

17. What is the discount of $150.75 for 1 year, at 6 per cent.? *Ans.* $8.533.

18. What is the present worth of $960, one half payable in 3 months, and in 6 months, and the rest in 9 months, discounting at 6 per cent.? *Ans.* $936.70.

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194. *The time, rate per cent., and interest being given, to find the principal.*

1. What sum of money, at 6 per cent, will, in 14 months, gain $14.? $1, at the given rate and time, will gain $ .07: hence,

Int. of $1.		Given int.		Princi'l of former.		Princi'l of latter.
$ .07	:	$14.	: :	$1.	:	$200., *Ans.*

From the above we easily deduce the following

RULE. Divide the given interest by the interest of ONE DOLLAR for the given rate and time.

2. If, in 16 months, at 6 per cent, a man pay $25.20 interest; what is the principal? *Ans.* $315.

3. What sum of money, will, in 18 months, gain $24.75, at 6 per cent? *Ans.* $275.

4. Received, for interest on a certain note, at the end of 1 year, $200; what was the principal, allowing interest at 6 per cent? *Ans.* $3333.3333⅓.

5. Sold a hogshead of molasses at 50 per cent. profit, and thereby gained $8.50; what did I give for it? *Ans.* $23.

6. If by selling calico at 20 per cent. loss, there is lost on each yard $ .08 what was the first cost? *Ans.* $. 20 *per yd.*

7. At the end of 1 year, a man received $4000. interest on a certain note, at the rate of 6 per cent.; what was the face of said note? *Ans.* $66666.66666⅔.

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195. *The principal, interest, and time being given, to find the rate per cent.*

1. A man received $16.33⅓ interest, for the use of $200 14 months, what was that per cent.?

The interest of the given sum, for the given time, at ONE *per cent.* would be $2.33⅓; therefore

Int. at 1 per cent.		Given int.		Per cent. of former.		Per cent. of latter.
$2.33⅓	:	$16.33⅓	: :	1	:	7 *per ct., Ans.*

Whence we have the following

RULE. Divide the given interest, by the interest of the given sum, for the given time, at ONE PER CENT.

2. If I pay $17.55 for the use of $234, 18 months, what is the rate per cent? *Ans.* 5.

3. If, for the use of $715, 2 years and 4 months, I pay $116.78⅔, what is that per cent.? *Ans.* 7.

4. If I pay $13.12½ for the use of $60, 1 year and 9 months, what do I pay per cent.? *Ans.* 12½.

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196. *The prices at which goods are bought and sold being given, to find the* GAIN OR LOSS *per cent.*

1. Bought cloth at $2.80 per yard, and sold it at $3.50; what did I gain per cent.?

This question does not differ essentially from the preceding. $3.50 − 2.80 = $ .70, gain on a yard.

Prime cost.		100 cents worth.		Gain by the former.		Gain by the latter.
$2.80	:	$1	: :	$ .70	:	$ .25 PER CENT., *Ans.*

As the second term will always be 1, we have this

RULE. Divide the GAIN OR LOSS by the PRIME COST.

2. If I buy coffee at 20 cents, and sell it at 24 cents, what do I gain per cent.? *Ans.* 20 *per cent.*

3. Bought a hogshead of rum containing 114 *gals.*, at 96 cents per gallon, and sold it again at $1.0032; what was the whole gain, and what the gain per cent.? *Ans. Whole gain* $4.924, *being* 4⅓ *per cent.*

4. A merchant bought a quantity of tea for $365, which proved to be damaged so that he could make but $332.15 of it; what did he lose per cent.? *Ans.* 9 *per cent.*

5. Bought a pipe of wine for $150, and sold it again for $1.25 per gallon; do I gain or lose, and how much per cent.? *Ans. I gain* 5 *per cent.*

6. Bought 30 hogsheads of molasses for $600; paid for duties, $20.66; freight, $40.78; cartage, $6.05, insurance, $30.84; if I sell them at $26 per hogshead, how much shall I gain per cent.? *Ans.* $11.695.

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197. *The principal, rate per cent., and interest being given, to find the time.*

1. In what time will $36 gain $3.78, at 7 per cent.

The interest for 1 year is $2.52; consequently,

Int. for 1 year.		Given interest.		Time of the former.		Time of the latter.
$2.52	:	$3.78	: :	1 year	:	1.5 *years, Ans.*

Hence we have this

RULE. Divide the GIVEN INTEREST by the interest of the given sum, at the given rate, for ONE YEAR.

2. Paid $31.71 interest on a note of $226.50; what was the time, the rate being 6 per cent.? *Ans.* 2.33⅓ = 2 Y. 4 *mo.*

3. In what time will $444.50 gain $133.35, at 8 per cent.? *Ans.* 3 Y. 9 *mo.*

4. The interest on a note of $999.999, at 5 per cent., was $397.499610416⅔; what was the time? *Ans.* 7 Y. 9 *mo.* 15 *d.*

198. We have seen, that the interest of any sum, for 1 year, is obtained *by multiplying it by the* rate per cent. In the

same way are calculated *commission, insurance, buying and selling stock, loss and gain,* or any thing else rated at so much per cent., WITHOUT REGARD TO TIME.

1. A man proposed to collect the taxes of a certain town, whose tax list amounted to $2525.04, for 2½ per cent.; how much will it come to?
Ans. $63.126.

2. A merchant, having purchased goods to the amount of $960, sold them so as to gain 12½ per cent.; for how much did he sell them?
Ans. $1080.

3. Bought goods to the amount of $575.62½; how must they be sold to gain 25 per cent.? *Ans.* $719.53⅛.

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199. COMMISSION is an allowance of so much per cent., to a person called a FACTOR, CORRESPONDENT, OR BROKER, for assisting merchants and others, in buying and selling goods.

4. A FACTOR buys and ships off to his employer in New York, goods to the amount of $1162; what will his commission come to at 2½ per cent.? *Ans.* $29.05.

5. My correspondent sends me word that he has purchased goods to the value of $1715.05, on my account; what will his commission come to at 3½ per cent.? *Ans.* $60.026.

6. A correspondent in London, buys and ships off to his employer in Philadelphia, goods to the amount of £1745 12*s.* 9*d.*; what is his commission at 5 per cent.? *Ans.* £87 5*s.* 7½*d.*

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200. INSURANCE is an exemption from hazard, obtained by the payment of a certain sum, which is generally so much PER CENT. on the value of the property insured.

201. PREMIUM is the sum paid by the *insured* for the insurance.

202. POLICY is the name given to the instrument of writing, by which the contract of indemnity is effected.

7. What will be the annual premium for insuring a house valued at $7900, at ¾ per cent.? *Ans.* $59.25.

8. A merchant sent out a ship and cargo, valued at $72500; what will be the premium at 4½ per cent.? *Ans.* $1812.50.

9. What will be the premium for insuring a cotton manufactory, valued at $50000, at 1½ per cent.? *Ans.* $750.

10. What will be the premium for insuring a ship and cargo, valued at $34500 at ¼ per cent.? —— ⅜ per cent.? —— ⅝ per cent.? —— ⅞ per cent.?
Ans. to the last, $215.62½.

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203. STOCK is a general name for the CAPITAL of any *corporation* or *trading company.*

The VALUE of stock is VARIABLE. When $100 of stock sells for $100 in money, it is said to be at PAR;* when for *more,* ABOVE PAR; when for *less,* BELOW PAR.

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* A Latin word, signifying *equal.*

11. What is the value of $2500 of stock, at 112½ per cent.; that is, 12½ per cent. *above* par, or 12½ per cent. *advance*, as it is sometimes called? *Ans.* $2812.50.

12. What is the value of $1500 bank stock, at 97½ per cent.; that is, at 2½ per cent. *below* par? *Ans.* $1462.50.

13. What is the value of $500 bank stock, at 95½ per cent.? —— at 88 per cent.? —— at 73½ per cent.? —— at 25 per cent. *below* par? —— at 107 per cent.? —— at 135 per cent.? —— at 62½ per cent. *advance*? *Ans. to last*, $812.50.

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204. LOSS AND GAIN. The several cases under this rule, which are generally found in arithmetics, fall under similar cases in interest. For, it is evident, that the purchase money, or as it is commonly called *prime cost* of goods, may be regarded as *principal;* gain or loss per cent., as *rate per cent.;* and the selling price, as the *amount*.

14. Bought tea at $ .75 per lb.; how must I sell it, to gain at the rate of 20 per cent.? *Ans.* $ .90 *a pound*.

15. Bought a hogshead of molasses for $23; how must I sell it per gallon so as to gain 50 per cent.? *Ans.* $ .50.

16. Bought a quantity of broadcloth at $2.50 per yard, which not proving so good as I expected, I am willing to lose 12½ per cent.; how must I sell it per yard? *Ans.* $2.18¾.

17. Bought a hogshead of sugar of 900lbs. for $60; how must I sell it by the lb. to gain 50 per cent.? *Ans.* $ .10.

18. Bought chocolate at $ .25 per pound, how must I sell it to gain 10 per cent.? *Ans.* $ .27½ *a pound*.

19. Bought calico at $ .20 per yard, how must I sell it to gain 4½ per cent.? —— 5 per cent.? —— 7 per cent.? —— 10 per cent.? —— 15 per cent.? —— 20 per cent.? *Ans. to last*, $ .24.

20. A man bought a piece of shalloon containing 34 yards, for £5 13*s*. 4*d*.; how must he sell it per yard to lose 12½ per cent.? *Ans.* 2*s*. 11*d*.

21. A merchant bought 315 bushels of wheat at 16*s*., and 537 bushels at 13*s*., and sold it, one with another, at 15*s*. a bushel; what did he gain on the whole, and what per cent.?

Ans. gained on the whole £37 19*s*., *per cent.* £6 6*s*. 3 $\frac{4185}{12021}$*d*.

NOTE. For other cases in *Loss and Gain*, see Arts. 191, 194 and 196.

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205. *To compute interest on notes, bonds, &c. when partial payments have been made.*

In Massachusetts the law provides, that payments shall be applied to keep down the interest, and that neither *interest* nor *payment* shall ever *draw interest*. Hence, if the payment at any time *exceeds* the interest computed to the *same* time, that excess is to be taken from the principal; but if the payment be less than the interest, the principal is to remain unaltered. Wherefore we have this

RULE. Compute the interest to the first time when a payment was made, which, either alone, or together with the preceding payments, if any, exceeds the interest then due; add that interest to the principal, and from the *sum* subtract the payments, or the *sum* of the payments, made within the time for which the interest was computed, and the remainder will be a new principal, with which proceed as with the first.

NOTE. The rule in New-York is somewhat more complex, but it amounts in the end to the same result, or *very nearly*.

1. *For value received I promise to pay* ALVAN STEWART, *or order, nine hundred ninety-nine dollars ninety-nine cents and nine mills, with interest. New-York, April* 1, 1823.
$999.999. SAMUEL REED.

On this note were the following endorsements:

Sept. 14, 1823, received	$166.666	In finding the *times* for computing interest, see Art. 107.
March 1, 1825, ...	$16.666	
July 17, 1826, ...	$50.000	
Aug. 5, 1827, ...	$333.333	
Dec. 29, 1827, ...	$621.000	

What was due Jan. 19, 1828?

The first principal on interest from April 1, 1823, ...	$999.999
Interest to Sept. 14, 1823, (5mo. 13d.,)	31.694
Amount	$1031.693
Payment, Sept. 14, exceeding interest then due, ...	166.666
Remainder for a new principal, (Sept. 14, 1823,)	865.027
Interest from Sept. 14, 1823, to Aug. 5, 1827, (46mo. 21d.) .	235.647
Amount	$1100.674
Payment, March 1, 1825, less than interest then due, $16.666	
Payment, July 17, 1826, less than interest then due, 50.000	
Payment, Aug. 5, 1827, exceeding interest then due, 333.333	
	$399.999
Remainder for a new principal, (Aug. 5, 1827,)	$700.675
Interest from Aug. 5, 1827, to Dec. 29, 1827, (4mo. 24d.,) .	19.618
Amount	$720.293
Payment, Dec. 29, 1827, exceeding interest then due, . .	621.000
Remainder for a new principal, (Dec. 29, 1827,)	$99.293
Interest due Jan. 19, 1828, from Dec. 29, 1827, (20 days,) .	.386
Balance due Jan. 19, 1828, (*Ans.*)	$99.679

2. *For value received I promise to pay* PETER PARLEY, *or order, eight hundred eighty-eight dollars eighty-eight cents and eight mills, with interest. New-York, Jan.* 17, 1820.
$888.888. EZRA CARY.

On this note were the following endorsements:

May 1, 1822, received	$155.555
Dec, 22, 1822, received	$25.555
Aug. 4, 1823, received	$30.000
Sept. 25, 1824, received	$52.222
July 17, 1827, received	$292.222

What was due Aug. 8, 1828? *Ans.* $345.702.

COMPOUND INTEREST.

208. COMPOUND INTEREST is that which arises from the principal and *interest*.

It is computed by adding the interest to the principal at the end of *each year*, and making the AMOUNT the principal for the *ensuing year*.

1. What is the compound interest of $275 for 3 years, at 6 per cent. ?

$ 275 given sum, or first principal.
.06

16.50 interest } to be added together for the
275.00 principal } principal of the next year.

291.50 *amount*, or principal for 2d year.
.06

17.4900 com. interest, 2d year, } to be added together.
291.50 principal, do. }

308.9900 *amount*, or principal for 3d year.
.06

18.5394 com. interest, 3d year, } to be added together.
308.99 principal, do. }

327.529 *amount*.
$275.... first principal subtracted.

Ans. $ 52.529 compound interest for 3 years.

2. What is the amount of $1, at 6 per cent., for 2 years? —— for 3 years? —— for 4 years? —— for 5 years? —— for 6 years? —— for 7 years? —— for 8 years? *Ans. to last*, $ 1.5938+

The amount of $2, for any given time, will be *twice* as much as the amount of $1; the amount of $3, will be *three times* as much, &c.

Hence, we may form the amounts of $1, for several years, into a table of *multipliers* for finding the amount of *any sum* for the same time.

TABLE,

Showing the amount of £1 or $1, &c. for any number of years, not exceeding 24, at the rates of 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1.05	1.06	13	1.88564	2.13292
2	1.1025	1.1236	14	1.97993	2.26090
3	1.15762	1.19101	15	2.07892	2.39655
4	1.21550	1.26247	16	2.18287	2.54035
5	1.27628	1.33822	17	2.29201	2.69277
6	1.34009	1.41851	18	2.40661	2.85433
7	1.40710	1.50363	19	2.52695	3.02559
8	1.47745	1.59384	20	2.65329	3.20713
9	1.55132	1.68947	21	2.78596	3.39956
10	1.62889	1.79084	22	2.92526	3.60353
11	1.71033	1.89829	23	3.07152	3.81974
12	1.79585	2.01219	24	3.22509	4.04893

If any principal be multiplied by the amount of $1, &c., as found above, it will evidently give the AMOUNT *of that principal*.

3. What is the amount, compound interest, of $500 for 3 years, at 5, and at 6 per cent.? *Ans.* $578.81 *at 5, and* $595.505 *at 6 per cent.?*

4. What is the amount of $629 for 7 years at 5 per cent. compound interest? Ans. $885.08

5. What is the compound interest of $1256 for 15 years, at 6 per cent.? *Ans.* $1764.0668.

6. What is the amount, compound interest, of £999 19s. 11¾d. for 7 years, at 5 per cent.? *Ans.* £1326 18s. 11d. 1.47304999952qrs.

THEORETICAL QUESTIONS.

What is interest? *How* is it computed? What is *rate per cent.*? What does *per cent. signify*? What is *principal*? —— *amount*? By what rule do you find the *interest* of any sum for 1 year? —— *for any number of years*? How do you find the *interest*, at 6 per cent., *for any given time*; that is for any number of years, months, and days? How, at 7 per cent.? How, at any OTHER RATE? How do you find the interest of any sum in *pounds*, &c.

The time, rate per cent., and amount being given, how do you find the principal? Having the prices at which goods are *sold*, and the gain or loss *per cent.*, how do you find the *prime cost*?* What is *discount*? —— *present worth*? —— the RULE? (See Rule, Art. 150.) The time, rate per cent., and interest being given, how do you find the principal? Having the gain or loss of goods, and the gain or loss *per cent.* how do you find the prime cost? (*The same rule as before, only in different language.*) The principal, interest, and time being given, how do you find the *rate per cent.*? The prices at which goods are *bought* and *sold* being given, how do you find the GAIN OR LOSS *per cent.*? The principal, rate per cent., and interest being given, how do you find the time?

How do you calculate *commission, insurance, buying and selling stock, loss and gain*, or any thing else rated at so much per cent. *without regard to time*? What is COMMISSION? —— INSURANCE? —— *premium*? —— *policy*? —— STOCK? What can you say with regard to the *value* of stock? When is it at *par*? —— above par? —— below par? What can you say of LOSS AND GAIN? What is the rule for computing interest on notes, bonds, &c., when partial payments have been made? What is the principle on which the rule is founded? What is compound interest? How is it computed?

EXAMPLES FOR PRACTICE.

1. What is the interest of $273.51 for 1 year 10 days, at 7 per cent.? *Ans.* $19.67.

2. What is the interest of $399.999 for 2 years, 9 months, and 19 days, at 8 per cent.? *Ans.* $89.68866 $\frac{8}{15}$

3. D's note of $333.17 was given Oct. 5, 1808, on interest after three months; Jan. 17, 1809, he paid $58; what was there due May 1, 1813?† *Ans.* $346.81828423

4. E's note of $777.777 was given Nov. 7, 1807, on interest after 90 days; Feb. 27, 1817, he paid $133.333; what was there due Dec. 17, 1827? *Ans.* $1571.29425

5. What will be the insurance of a ship and cargo, valued at $9693, at ½ per cent.? —— at ¼ per cent.? —— at $\frac{7}{16}$ per cent.? at ¾ per cent. *Ans. to last*, $72.69.75

6. What is the value of $700 United States Bank stock, at 112½ per cent.? *Ans.* $787.50

7. What principal will, in 9 months 18 days, at 7 per cent., amount to $542.78½? *Ans.* $514.

8. What is the value of $500.75 stock, at 93 per cent.? *Ans.* 521.497.

9. A man compromised with his creditors at 62½ cents on a dollar: what then must he pay on a debt of $555.46? *Ans.* $347.1615.

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* give the process in his own language, as exhibited under

† Interest at 6 per cent.

10. Bought a house and lot for $6000, payable in 9 months, which I sold again for $5500, payable in 15 months; what did I gain on a hundred, the usual rate being 6 per cent.? *Ans.* $1.65795.

11. Sold goods for $3120, to be paid, one half in 3 months, and the other half in 6 months; what must be discounted for present payment?
Ans. $3068.492.

12. The interest on a certain note, for one year 9 months, was $49.575 what was the principal? *Ans.* $475.

13. Bought 21 barrels of tobacco, weighing 3453*lbs.*, at $9.25 for 100*lbs.* ready money, and sold it again at $[illegible]87½ per hundred on 4 months credit; what did I gain per cent.? *Ans.* 60.26\frac{11}{17}$

14. A merchant bought 10 tons of iron for $950; the freight and duties came to $145, and his own charges to $25, how must he sell it per pound to gain 20 per cent.? *Ans.* 6 *cents.*

15. Bought cloth at 15*s.* per yard, which not proving so good as I expected, I am content to lose 17½ per cent.; how must I sell it per yard?
Ans. 12*s.* 4$\frac{4}{7}$*d.*

16. A stationer sold quills at 8*s.* 9*d.* per hundred, by which he gained $\frac{1}{8}$ of his money, he afterwards found them to be in great demand and raised them to 9*s.* 6*d.* per hundred; what did he gain per cent. by the latter price?
Ans. £26 12*s.* 4*d.*

17. Bought a pipe of wine containing 126 gallons at 10*s.* per gallon, but by accident 16 gallons leaked out; at what rate must I sell the remainder per gallon, to gain upon the whole 12½ per cent.?
Ans. 12*s.* 10$\frac{7}{11}$*d.*

18. Suppose I receive from Richmond, Virginia, 12 hogsheads of tobacco, weighing 15457*lbs.*, which, with the charges there, amounted to $796.85; freight from thence to New-York $6 per hogshead, cartage and storage there 90 cents per hogshead, and freight to Albany $1.50 per hogshead; what does it stand me in per hundred, and what will be my gain per cent. by selling it at 7½ cents a pound on 5 months credit?

Ans. { It stands me in 5.80\frac{11446}{12417}$ *per h.*
{ My gain per cent. is 25.99\frac{4112273}{7386713}$.

EQUATION OF PAYMENTS.

207. Equation of payments is a method of finding the mean time for the payment of several sums, due at different times.

I. Suppose I have two notes against a man; one of $6, due in 2 months, the other of $4, due in 5 months; and he wishes to pay both of them at one time; what time ought he to have?

$6 for 2 months is the same as $1 for 12 months, &
$4 for 5 months is the same as $1 for 20 months.

10 — 32

Hence it is obvious, that he may keep *one dollar* 32 months; consequently he may keep 10 dollars $\frac{1}{10}$ part of the time. $\frac{1}{10}$ part of 32 is (32 ÷ 10 =) 3mo. 6d., *Answer.* Hence,

208. *To find the mean time of several payments*, we have this

RULE. Multiply each sum by its TIME of payment, and divide the sum of the PRODUCTS by the sum of the PAYMENTS.

The rule is founded on the supposition that what is gained by keeping a debt a certain time *after* it is due, is equal to what is lost by paying it a certain time

fore it is due; but, in the first case, the gain is equal to the interest of the debt for the given time, and in the second, the loss is only equal to the *discount* of it for the same time; which is always LESS *than the interest*. The rule, therefore is not exactly true; but the errour is too small to be regarded in business.

2. A owes B $200, of which $40 is to be paid in 3 months, $60 in 5 months, and the remainder in 10 months; what is the equated time for the payment of the whole? *Ans.* 7*mo.* 3*d.*

3. A man bought a quantity of goods, for which he is to pay ¼ part cash, ¼ in 3 months, and the remaining ½ in 8 months; but it was afterwards agreed to pay the whole at one equated time; required that time? *Ans.* 4⅜ *months.*

4. C owes D $1400, to be paid in 3 months, but D being in want of money, C pays him at the expiration of 2 months, $1000; how much longer than 3 months ought C, in equity, to defer the payment of the rest? *Ans.* 2½ *months.*

5. Peter is indebted to William $164.166, ⅓ of which is to be paid in 6 months, ¼ in 8 months, and ⅙ in 12 months; I demand the equated time for the payment of the whole? *Ans.* 7½ *months.*

THEORETICAL QUESTIONS.—What is equation of payments? What the RULE? Upon what is the rule founded? Wherein is it not exactly true?

FELLOWSHIP.

209. Fellowship is a method of ascertaining the respective gains or losses of individuals, engaged in joint trade.

210. The money, or value of the articles employed, is called the *Capital* or *Stock*; the gain or loss to be shared is called the *Dividend.*

Two men bought goods; A paid $60, and B $40; in the sale of them they lose $20; what share of the loss must each man sustain?

211. It is evident that *each man's* gain or loss ought to have the same ratio to the whole gain or loss, as his *share of the stock* to the *whole stock.* Hence we have this

RULE. As the WHOLE stock: is to each man's SHARE of the stock: : so is the WHOLE gain or loss: to his ***share*** of the gain or loss.

2. Two men are partners in trade; A put in $350, and B $450; they gain $500; what is each man's share of the gain?

The whole stock is 350 + 450 = $800; Then

Whole stock.	Each man's stock.	Whole gain.	Each man's share of gain.
As $800 :	$350	: : $500 :	$218.75 A's gain.
	$450		$281.25 B's gain.

Proof $500.00 whole gain.

3. Two boys, Charles and Henry, bought a lottery ticket; Charles paid $1.50, and Henry $2; the ticket drew a $20 prize; how much of it is each one to receive? *Ans.* Charles $8.571, and Henry 11.42.

4. Two persons have a joint stock of $1000; whereof A put in $450, and B 550; by ill luck they lose $350: what share of the loss must each sustain? *Ans.* A's $157.50. B's 192.50.

5. A, B and C bought a ticket for $3; A paid $ .90, B $1, and C the remainder; the ticket drew a prize of $2000, subject to a deduction of 12½ per cent.; how much is each to receive?

Ans. A $525, B $583.333⅓, and C $641.666⅔.

trade in company; A's capital was $275, B's $300, and

C's $525; by misfortune they lose $400; what share of the loss must each sustain?

Ans. { A must lose $100. B..... $109.09 1/11. C..... $190.90 10/11.

7. Divide $240 among 3 persons, so that their shares shall be to one another as 1, 2, and 3, respectively. *Ans.* $40, $80, and $120.

8. A, B, C and D companied; A put in $150, B $250, C $275, and D $325; they gained $337.50; what was each man's share?

Ans. { A's share $ 50.62½. B's ..... $ 84.37½. C's ..... $ 92.81¼. D's ..... $109.68¾.

9. Three persons A, B and C freighted a ship with 340 tuns of wine; of which A loaded 110 tuns, B 97, and C the rest; in a storm, the seamen were obliged to throw overboard 85 tuns; how much must each sustain of the loss? *Ans. A must lose* 27½ *tuns*, B 24¼, *and* C 33¼.

10. A and B companied; A put in $75, and took $\frac{3}{5}$ of the gain; how much did B put in?

Their respective shares of the gain are to be in proportion to each man's stock; hence if 3 fifths of the stock be $75, 2 fifths of it would be $50. *Ans.*

11. Two men trade in company with a capital of $1000; they gain a certain sum, whereof A's share is ½ as much as B's; what was each man's stock? *Ans.* A's $333.33⅓. B's $666.66⅔.

12. Four merchants, A, B, C and D built a sloop which cost them $1730; the freight for her first voyage amounted to $370, of which A's share was $74, B's 111, C's $148, and D's $27; what was each man's stock?

Ans. A's *stock was* $346, B's $519, C's $692, *and* D's $173.

13. A, B and C trade in company; A put in $40, B and C together $170; they gain $126, of which B takes $42; what did A & C gain, and B and C put in respectively?

Ans. A *gained* $24, C 60; B *put in* $70, *and* C 100.

14. A bankrupt is indebted to A $120, to B $230, to C $340, and to D $450; his whole estate is worth only $560; how must it be divided among the creditors?

Ans. $58.947\frac{42}{114}$ *to* A, $112.982\frac{52}{114}$ *to* B, $167.017\frac{62}{114}$ *to* C, *and* $221.052\frac{72}{114}$ *to* D.

15. If a cotton factory, valued at $40000, be divided into 100 shares, and the profits amount to 15 per cent. yearly; what will be the profit on 1 share? —— 2 shares? —— 3 shares? —— 5 shares? 15 shares? —— 30 shares? *Ans. to last*, $1800.

16. If in the above-mentioned factory, repairs are to be made which will cost $750; what will be the tax on each share? —— on 5 shares? —— on 11 shares? —— on 15 shares? —— on 20 shares?

Ans. to last, $150.

17. If a certain town raise a tax of $1710, and the whole town be valued at $57000, what will be the tax on $1? —— on an estate valued at $2300? *Ans. on* $1 *it will be* $ .03, *and on* $2300 $69.

212. In assessing taxes, an inventory of the property of the whole town, both *real* and *personal*, and also of the whole number of polls, is first to be obtained. And, if any part of the tax is to be averaged on the polls, or otherwise, we must first deduct what the *polls*, &c. amount to; and then *with the remainder*, we may find the tax on $1, and by it make a table containing the taxes on 1, 2, 3, &c., to $10; then on 20, 30, &c., to $100; and then on 200, 300, &c. to $1000. Then, knowing the inventory of an individual, it will be easy to find the tax on his property.

18. A certain town, valued at $94960, raises a tax of $2343; there

740 polls, which are taxed $ .60 apiece; what is the tax on a dollar, and what will be A's tax, whose *real estate* is valued at $1970, personal property at $825, and who pays for 3 polls?

740 polls at $ .60 = $444, amount of the poll taxes, and $2343 — $444 = $1899; then $94950 : $1899 : : $1 : $ .02, tax on $1.

TABLE.

	dolls.	dolls.		dolls.	dolls.		dolls.	dolls.
Tax on	1	is .02	Tax on	10	is .20	Tax on	100	is 2.
" "	2	" .04	" "	20	" .40	" "	200	" 4.
" "	3	" .06	" "	30	" .60	" "	300	" 6.
" "	4	" .08	" "	40	" .80	" "	400	" 8.
" "	5	" .10	" "	50	" 1.00	" "	500	" 10.
" "	6	" .12	" "	60	" 1.20	" "	600	" 12.
" "	7	" .14	" "	70	" 1.40	" "	700	" 14.
" "	8	" .16	" "	80	" 1.60	" "	800	" 16.
" "	9	" .18	" "	90	" 1.80	" "	900	" 18.

The tax on 1000 dollars is 20 dollars.

Now to find A's tax, his real estate being $1970, we find by the table, that

The tax on	$1000	is $20.
Do. on	900	" 18.
Do. on	70	" 1.40
Tax on his real estate		$39.40
In like manner we may find the tax on his personal property to be		16.50
3 polls at $ .60 each,		1.80
Tax required,		$57.70

19. What will be the tax of a man, whose inventory is 987 dollars *real*, and 540 dollars *personal* property, and who pays for 4 polls? *Ans.* $32.94

213. DOUBLE FELLOWSHIP, OR FELLOWSHIP WITH TIME.

1. Two men hire a pasture for $9; A put in 6 cows for 3 months, and B 3 cows for 4 months; how much ought each man to pay?

6 cows for 3 months is the same as 18 cows for 1 month.
3 cows for 4 months is the same as 12 cows for 1 month.

The shares of A and B are, therefore, as 18 to 12; that is, as the product of each man's stock into the time he continues it in trade. Wherefore we have this

214. RULE. ***Multiply each man's stock by the time he continues it in trade, then say, as the sum of the products : is to each particular product : : so is the whole gain or loss : to each man's*** SHARE ***of the gain or loss.***

2. Three merchants, A, B and C, enter into partnership; A puts in $240 for 4 months, and B $120 for 6 months, and C $200 for 8 months; they gain $260; what is each man's share of the gain?

Ans. { *A's gain* $ 76.09$\frac{1}{11}$. / *B's* " $ 57.07$\frac{7}{11}$. / *C's* " 126.82\frac{9}{11}$. }

3. ...r into partnership for 16 months; A put in at first $1200, ... months $200 more; B put in at first $1500, and at the

expiration of 6 months took out $500;—they gained $772.20; what is each man's part? *Ans.* *A's* $401.70—*B's* $370.50.

4. A ship's company take a prize, value £4000, which they agree to divide among them, according to their pay and the time they have been on board; the officers and midshipmen have been on board 4 months, and the sailors 3; the officers have 50*s.* a month, the midshipmen 40, and the sailors 27; moreover there were 4 officers, 8 midshipmen, and 120 sailors; what share of the prize must each person have?

Ans. { £67 15*s.* 11*d.* $0\frac{22}{29}$*qrs.*, *each officer.*
{ £54 4*s.* 8*d.* $3\frac{25}{29}$*qrs.*, *each midshipman.*
{ £27 9*s.* 1*d.* $3\frac{25}{29}$*qrs.*, *each sailor.*

THEORETICAL QUESTIONS.

What is fellowship? What is the money, or value of the articles employed, called? What the gain or loss to be shared? What is the RULE? What is the method of assessing taxes? What is double fellowship? What the RULE?

ALLIGATION

215. Is a method of mixing two or more simples of different quantities, so that the composition may be of a mean or middle quality.

216. When the QUANTITIES and PRICES of the simples are given, to find the MEAN PRICE of the mixture, it is called ALLIGATION MEDIAL.

1. A grocer mixed together 4lbs. of sugar, worth 10 cents a lb., 5lbs. worth 12 cents a lb., and 10lbs. worth 14 cents a lb.; what is a pound of the mixture worth?

217. It is evident that the price of the WHOLE, divided by the QUANTITY, will give the PRICE OF ONE.

4 lbs. at 10 cents, cost	40 cents.		
5 lbs. at 12 cents, cost	60 cents.	*lbs.*	*cents.*
10 lbs. at 14 cents, cost	140 cents.	Then 19)	240 (*gives* $12\frac{12}{19}$, *Ans.*
19 lbs. cost . . .	240 cents.		

2. A farmer would mix 20 bushels of wheat at $1 per bushel, 16 bushels of rye at $ .75 per bushel, 12 bushels of barley at $ .50 per bushel, and 8 bushels of oats at 40 cents per bushel; what will a bushel of the mixture be worth? *Ans.* $ .735$\frac{5}{7}$

3. A grocer puts 9 gallons of water into a cask containing 54 gallons of rum, worth 50 cents a gallon; what is a gallon of the mixture worth? *Ans.* $ .428$\frac{4}{7}$

4. A goldsmith melted together 3 ounces of gold of 20 carats fine, and 5 ounces of 22 carats fine; what is the fineness of the mixture? *Ans.* 21$\frac{1}{4}$ *carats.*

5. A merchant mixes 12 gallons of wine, at 75 cents per gallon, with 24 gallons at 90 cents, and 16 gallons at 110 cents; what is a gallon of the mixture worth? *Ans.* $ .926

6. Five hours of a certain day, the mercury was observed to stand in the thermometer, at 64°, 4 hours at 70°, 2 hours at 75°, and 3 hours at 73 degrees; what was the *mean* temperature for that day?

Ans. 69$\frac{3}{14}$.

218. *When the prices of* SEVERAL SIMPLES, *and the* MEAN PRICE *are given, to find what quantity of each must be taken, it is called* ALLIGATION ALTERNATE.

Hence it is evident, that Alligation Alternate is the reverse of Alligation Medial, and may be proved by it.

A grocer has sugar at 10 cents and at 17 cents a pound, which he would mix, so as to sell the mixture at 13 cents a pound; what quantity of each must he take?

Now, if the price of the mixture had *exceeded* the price of the first sort JUST AS MUCH as it *fell short* of that of the other, it is evident, he must have taken *an equal quantity of* EACH; had it exceeded the price of the first sort by only *one half as much* as it fell short of that of the other, it is plain, he must have taken TWICE *as much of the* ONE *as of the* OTHER. Hence, in all cases, the LESS the difference between the price of the *mixture* and *either* simple, the GREATER must be the quantity of *that simple*, in proportion to the *other*; that is, the quantities of the simples must be *inversely* as the *differences* between their prices and the price of the mixture: consequently, if these simples *be mutually* EXCHANGED, *they will*, DIRECTLY, *express the* RELATIVE QUANTITIES *of each simple necessary to form the compound required.*

In the above example, the price of the mixture is 13 cents, and that of the first simple 10 cents; the difference between them is 3 cents. The price of the other simple is 17 cents, which differs from the price of the mixture by 4 cents. By exchanging these differences, we have 4lbs. at 10 cents a pound, to 3lbs. at 17 cents, for the *Ans.*

We may now prove this result by the last rule.

4lbs. at 10 cents, cost	40 cents	
3lbs. at 17	51	Then 7)91(gives 13 cents, for the price of the *mixture*.
—	—	
7lbs. cost - - - - - -	91 cents.	

It is obvious, that what has been observed of *two* simples will apply to any number of PAIRS, if in EACH the price of *one* simple is GREATER, and that of the *other* LESS than the *mean price*. Hence we have this

219. RULE I. Connect with a continued line, each price that is LESS than the mean rate, with *one or more* that is GREATER, and each price GREATER than the mean rate, with one or more that is LESS.

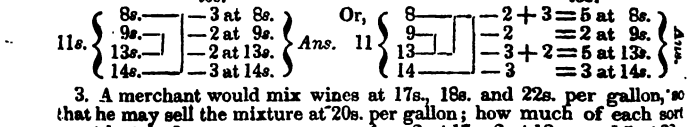
II. Write the difference between the mean rate or price, and the price of each simple, opposite the price with which it is CONNECTED*; then the sum of the differences standing against any price, will express the relative quantity to be taken of that price.

NOTE 1. The mean rate, or price of the mixture, and the prices of the several simples, must first be reduced to the same denomination.

2. It is evident from the rule, that there may be as many different answers as there are different ways of linking the several prices.

2. A merchant would mix tea, at 8s., 9s., 13s., and 14s. a pound, so that the mixture may be worth 11s. a pound; how much of each must he take?

OPERATIONS.



3. A merchant would mix wines at 17s., 18s. and 22s. per gallon, so that he may sell the mixture at 20s. per gallon; how much of each sort must he take? *Ans.* 2 *at* 17s., 2 *at* 18s., *and* 5 *at* 22s.

4. A goldsmith would mix gold of 18 carats fine, with that of 23, of 19, and of 16 carats fine, so that the composition may be 20 carats fine; what

* In this way, the difference between the two prices in each pair and the mean *rate, will* be mutually exchanged, so that there will be just as much *gained* by the *one as is lost* by the other, and therefore the gain and loss, upon the whole will be *equal.*

quantity of each must he take? *Ans.* 3 *each of* 16, 18, 19, *and* 7 *of* 23.

5. How much oats at 2s. 6d., barley at 3s. 8d., corn at 4s., and rye at 4s. 8d. per bushel, must be mixed together, that the compound may be worth 3s. 10d. per bushel?

Ans. 2 *bushels each, of barley and corn,* 10 *of oats, and* 16 *of rye.*

6. I desire to know how much gun-powder at 36cts., 27cts., and 24cts. a pound, will compose a mixture worth 30cts. a pound?

Ans. 9*lbs. at* 36*cts., and* 6*lbs. each at* 27*cts. &* 24*cts.*

7. What quantities of sugar, at 8cts., 10cts., and 14cts. a pound, will compose a mixture worth 12cts. a pound?

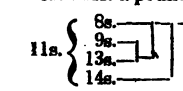
Ans. 2*lbs. at* 8 *&* 10*cts., and* 5*lbs. at* 14 *cents.*

220. As the quantities found by the rule, only express the RELATIONS of each kind, it is evident, that a compound of the *same mean price* would be formed by taking 2 *times*, 3 *times*, *one half*, *one-third*, &c. of each quantity so found. Hence,

When the quantity of one simple is given,

RULE. First, find the RELATIVE QUANTITY by the foregoing rule, then say, as the difference standing against that simple, WHOSE QUANTITY IS GIVEN : is to THAT QUANTITY : : so is each of the other DIFFERENCES, or relative quantities : to the REQUIRED quantities of each.

1. A merchant has 9lbs. of tea at 14s. a pound, which he would mix with other teas at 8s., 9s., and 13s. a pound, so that the mixture may be worth 11s. a pound; what quantities of each must he take?



Then 3 : 9 : : lbs. lbs. 5 : 15 at 8s. / 2 : 6 at 9s. / 5 : 15 at 13s. *Ans.*

2. A merchant would mix wines at 6s., 8s., and 11s. per gallon, with 49 gallons at 15s. per gallon, so that the mixture may be worth 13s. per gallon; how much of each must he take? *Ans.* 7 *gallons.*

3. A grocer has sugar worth 7cts., 9cts., and 12cts. a pound, which he would mix with 20lbs. at 14cts., that the mixture may be worth 10cts. a pound; what quantities of each must he take?

Ans. 26$\frac{1}{4}$*lbs. at* 7*cts.*, 13$\frac{1}{2}$*lbs. at* 9*cts., and* 6$\frac{3}{4}$*lbs. at* 12*cts.*

4. How much tea at 6s. 6d., 7s. 6d., and 9s. a pound, must be taken to mix with 36lbs. at 12s. a pound, that the mixture may be worth 8s. a pound? *Ans.* 108*lbs. at* 9*s.*, 288*lbs. at* 7*s.* 6*d., and* 72*lbs. at* 6*s.* 6*d.*

5. How much water of no value, must be mixed with 63 gallons of brandy at $1.30 per gallon, that the mixture may be worth $1.08 per gallon? *Ans.* 12$\frac{6}{8}$ *gallons.*

6. How much brass at 28cts., and pewter at 21cts. a pound, must I melt with 50lbs. of copper at 32cts. a pound, so that the mixture may be worth 24cts. a pound? *Ans* 50*lbs. at* 28*cts., and* 200*lbs. at* 21*cts.*

7. An innkeeper would, with 72 bushels of his best grain, at 80cts. a bushel, mix other grain at 64cts., at 48cts., and at 32cts. a bushel; what quantities of each must he take, that the mixture may be worth 56cts. a bushel?

Ans.
- 24 *bushels each at* 64*cts. and* 48*cts., and* 72 *bushels at* 32*cts.*
- 216 *bushels each at* 64..... *and* 48*cts., and* 72 *bushels at* 32*cts.*
- 18*bush. at* 64, 72*bush. at* 48, *and* 54*bush. at* 32*cts.*
- 96..... *at* 64, 24 ..... *at* 48, *and* 96..... *at* 32*cts.*
- 54 *bushels each at* 64 *and* 48*cts., and* 72*bush. at* 32*cts.*
- 288 .... *at* 64 *and* 48*cts., and* 72..... *at* 32*cts.*
- 72 *bushels of each sort.*

* Which stands against the price whose quantity is given.

8. A merchant has 6lbs. of tea at 9s. a pound, and 9lbs. at 14s., which he would mix with some at 8s., and some at 13s.; how much of these two last must he take that the mixture may be worth 11s. a pound?

It is obvious, that we *first find the mean rate of the simples when quantities are given*, and then proceed as before. *Ans.* 15*lbs. of each.*

9. How much gold of 15 and 17 carats fine, must be mixed with 5oz. of 18, and 13oz. of 22 carats fine, that the compound may be 10 carats fine? *Ans.* 2*oz. of each.*

10. How much wine at 80cts. and at 87½cts. per gallon, must be mixed with 8 gallons at 75cts, and 12 gallons at 90cts. a gallon, that the mixture may be worth 82½cts. a gallon?

Ans. 52 *gallons at* 80*cts., and* 20 *gallons at* 87½

221. *When the quantity of the compound is given,*

RULE. As the sum of the RELATIVE QUANTITIES, found by the FOREGOING RULE : is to the quantity REQUIRED : : so is each QUANTITY SO FOUND : to the REQUIRED quantities of EACH.

1. A merchant has tobacco at 10cts., at 12cts., and at 14cts. a pound, of which he would sell A 100lbs. at 13cts. a pound; how much of each kind must he take?

We find the relative quantities to be 3lbs. at 10cts., 3lbs. at 12cts. and 4lbs. at 16cts.; the sum of which is $3+3+4=10$lbs.

	lbs.		*lbs.*			
Hence,	10	:	100	: :	3*lbs.* : 30*lbs. at* 10*cts.* 3*lbs.* : 30*lbs. at* 12*cts.* 4*lbs.* : 40*lbs. at* 16*cts.*	*Ans.*

2. A grocer has currants at 8d., 12d., 18d. and 22d. a pound; the poorest will not sell, and the best are too dear; he therefore concludes to make a mixture of 120lbs., and so much of each sort as to sell the mixture at 16d. a pound; how much of each must he take?

Ans. 36*lbs. at* 8*d.*, 12*lbs. at* 12*d.*, 24*lbs. at* 18*d.*, *and* 48*lbs. at* 22*d.*

3. A retailer has by him four sorts of tea, viz., at 5s., 6s., 8s. and 9s. a pound; out of these he is ordered to make up a chest, containing 168lbs., so as to be worth 7s. a pound: what quantity of each must he take?

4. A merchant would form a compound of 72 gallons of wine, of some at $.75, $1.25, $1.50, and $1.62½ a gallon, so as to sell it at $1.37½; what quantities of each must he take?

Ans. 8 *gallons at* $.75, 16 *at* $1.25, 40 *at* $1.50, *and* 8 *at* $1.62½

5. A silversmith has silver of 11½oz. fine, and of 7oz. fine, and has orders to make up a piece of work, requiring 35lbs. of 9½oz. fine; how much of each must he take. *Ans.* 21*lbs.* 4*oz.* 15*part.* 5*grs.* of 11½*oz. fine*; *and* 13*lbs.* 7*oz.* 6*part.* 16*grs. of* 7*oz. fine.*

6. Suppose 9lbs. of pure gold immersed in a vessel full of water to expel 3lbs. of water, 9lbs. of pure silver to expel 6lbs, and 9lbs. of a mass made up of gold and silver, to expel 4lbs.; required the quantities of gold and silver in said mass? *Ans.* 6*lbs. of pure gold, and* 3*lbs. of silver.*

7. Hence is solved the curious question concerning King Hiero's crown.—Hiero, king of Sicily, ordered a crown to be made containing 63oz. of pure gold: but suspecting the workmen had debased it by substituting part silver therein, he recommended the detection of the fraud to the famous Archimedes, who putting it into a vessel of water found it raised the fluid, or that itself contained 5.2245 cubic inches of metal, and having discovered that the cubic inch of gold more critically weighed 10.36 ounces, and that of silver but 5.85 ounces, he found by calculation what part of his Majesty's gold had been changed: and you are desired to do the same.

Ans. 24 1864*oz. of pure gold, and* 25.3936*oz. of silver.*

THEORETICAL QUESTIONS.

What is alligation? —— alligation medial? —— the rule? What is alligation alternate? When the price of the mixture, and the price of the several simples, are given, how do you find the relative quantities of EACH? When the quantity of one simple is given, how do you find the others? When the quantity of the whole compound is given, how do you find the quantity of EACH SIMPLE?

POSITION.

222. Position is a rule, that by FALSE or SUPPOSED numbers, taken at pleasure, discovers the TRUE one required.

It is of two kinds, SINGLE and DOUBLE.

SINGLE POSITION.

223. SINGLE POSITION is that which makes use of but ONE supposed number.

* * * * * * * * * * * * * *

224. RULE.* Take ANY convenient number, and proceed with it as though it were the *true one;* then say, as the RESULT is to the GIVEN SUM :: so is the *supposed number:* to the *number required.*

1. A person being asked how old he was, answered, "if to my age you add $\frac{1}{3}$ of it, and $\frac{1}{4}$ of it, the sum will be 44;" required his age?

Suppose his age was 18 years; then
$\frac{1}{3}$ of it, would be 6 .....
$\frac{1}{4}$ of it, ... 9 .....

	Given Sum.		Sup'd. Number.		Number Req'd.	
Then, as the RESULT	33	:	44	::	18	: 24, *Ans.*

BETTER ANALYTICALLY. $\frac{1}{3}$ of his age *and* $\frac{1}{4}$ of his age $= \frac{7}{12}$ of his age; this added to his age by the question is equal to 44; that is, $\frac{5}{6} + \frac{6}{6} (= \frac{11}{6}) = 44$. Hence $44 \div \frac{11}{6} = 24$ years, *Ans.*

2. A school-master, being asked how many scholars he had, replied, "if $\frac{2}{3}$ of the number I have be multiplied by 7, and $\frac{5}{7}$ of this product be divided by 3, the quotient will be 20;" what number of scholars had he? *Ans.* 30.

The same analytically. If the quotient by dividing by 3 is 20, the dividend is $3 \times 20 = 60$; this by the question, is equal to $\frac{5}{7}$ of the product of $\frac{2}{3}$ of the numbers multiplied by 7; consequently, $60 \div \frac{5}{7} = 84 =$ the product of $\frac{2}{3}$ of the number $\times$ by 7; and $84 \div 7 = 12 = \frac{2}{3}$ of the number. Hence $12 \div \frac{2}{3} = 30$, *Ans.*

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* Questions that belong to this rule are such, that the number sought is multiplied or divided by some known number; or is increased or diminished by *itself* a certain number of times. Consequently the results will be proportional to the suppositions; and hence the reason of the rule is obvious:

	Result.		Result.		Nos. from which each result is obtained.		
Thus,	12	:	36	::	12	:	6
And	12×3	:	36×3	::	2	:	6
	$12 \times \frac{1}{3}$	:	$36 \times \frac{1}{3}$	::	2	:	6
	$12 \pm \frac{12}{3}$	:	$36 \pm \frac{36}{3}$	::	2	:	6

3. A vessel has 3 pipes; the first can fill it in ½ an hour, the second in ⅓ of an hour, and the third in ¼ of an hour; in what time will all three fill it together? *Ans.* $\frac{1}{9}$ *hour.*

By analysis. It is obvious, that the first will fill 2 in 1 hour, that the second will fill 3 in one hour, and that the third will fill 4 in one hour; Consequently all three will fill 2 + 3 + 4 = 9 in 1 hour; hence they will all fill 1 in $\frac{1}{9}$ of an hour, *Ans.*

4. A gentleman distributed 78 cents among a number of poor people, consisting of men, women, and children; to each man he gave 6 cents, to each woman 4, and to each child 2; now there being twice as many women as men, and thrice as many children as women, it is required to find the number of each? *Ans.* 3 *men*, 6 *women, and* 18 *children.*

5. A, B & C, talking of their ages, B said his age was once and a half the age of A, B said his was twice, and one tenth the age of both, and that the sum of their ages was 98; what was the age of each?
Ans. A's 12, B's 18, *and* C's 63.

6. There is a cistern with 3 unequal pipes; if the largest pipe be opened it will be empty in 1 hour; if the second be opened it will be empty in 2 hours; if the third be opened it will be empty in 3 hours: required the time it will take to empty if all run together? *Ans.* $32\frac{8}{11}$ *minutes.*

7. A, B, C and D spent 35*s.* at a reckoning, and, being a little dipped, they agreed that A should pay ⅔, B ½, C ¼, and D ⅛: what must each pay in such proportion? *Ans.* A 13*s.* 4*d.*, B 16*s.*, C 6*s.* 8*d.*, *and* D 5*s.*

8. Peter drinks a barrel of beer in 24 days, Charles when he goes about it, does it in 16 days; now if they should both drink together, in what time would they finish it? *Ans.* 9*d.* 14*h.* 24*m.*

9. A gentleman bought a chaise, horse and harness for $500; the horse cost ¼ more than the harness, and the chaise ⅓ more that the horse; what was the price of each?
Ans. chaise 212.765\frac{15}{17}$, *horse* 159.574\frac{23}{47}$,
harness 127.659\frac{27}{47}$.

10. A wolf says he can eat a sheep in 60 minutes, a bear says he can eat it in half the time, a lion says he can eat it as quick as both; in what time can all three eat it together? *Ans.* 10 *minutes.*

11. A gentleman divided his fortune among his sons, to A he gave $9 as often as to B $5, and to C but $3 as often as to B $7, yet C's portion came to $1059; what was the whole estate? *Ans.* $7979.80.

DOUBLE POSITION

225. Is that which makes use of TWO supposed numbers.

* * * * * * * * * * * * * *

226. RULE* I. Take ANY TWO convenient numbers, and proceed with EACH as though it was the TRUE ONE required.

II. Place the RESULTS or ERRORS against their *positions* or *supposed numbers*; and if *any errour* be TOO GREAT, mark it with +; but if TOO SMALL with —.

III. Multiply them crosswise; that is, the *first position* by the *last errour*, and the *last position* by the *first errour*.

IV. Then if the errours are ALIKE, divide the DIFFERENCE *of*

* The rule is founded on this supposition, that the first errour is to the second as the difference between the true and first supposed number, is to the difference between the second supposed number. Where this is not the case, therefore, the [illegible] to the question cannot be found by this rule.

the products by the **DIFFERENCE** ***of the errours;*** but if the errours are **UNLIKE**, divide the **SUM** ***of the products*** by the **SUM** ***of the errours.***

1. B asked C how much his horse cost; C replied, that if he *cost* him 3 times as much as he did and 15 dollars more, he would stand him in $300; what was the price of the horse?

1st. Suppose he cost $90	2. Suppose he cost $96
3 times	3
As much = 270,	288
And $15 more	15
Is $285 *too little* by $15.	$303 *too much* by $7

90 × 15 — 96 × 3 +

15	1440	270	285
3	270		3
			15
18)	1710(95, *Ans.*		$200

Better analytically. It is obvious, that, taking the question backwards, 300 — 15 ÷ 3 will give the cost of the horse. 300 — 15 = 285, and 285 ÷ 3 = 95, *Answer, as before.*

2. A and B have the same income; A saves ⅓ of his, but B, by spending $75 per annum more than A, at the end of 8 years finds himself $100 in debt; what is their income, and what does each spend yearly?

Ans. their income is $500; A *spends* $437½, *and* B $512½.

The same by analysis. On an average, it is evident, that B falls in debt 12½ dollars every year; by the question, therefore, it is plain that ⅓ of their income + $75 is equal to ⅔ of it + $12½; consequently, ⅓ of their income is equal to $75 — 12½ ($62½.) Hence their income is 8 times $62½ = $500, *Answer, as before.*

3. A man was hired for 50 days on these conditions, that for every day he worked, he should receive $ .75, and, for every day he was idle, he should forfeit $ .25; at the expiration of the time, he received $27.50; how many days did he work, and how many was he idle?

Ans. he worked 40 *days, and was idle* 10.

By analysis. Had he worked the whole time, his wages would evidently have been $ .75 × 50 = $37.50, which is $10 more than he did receive; but every day he was idle lessened his wages $75 + $25 = $1; consequently he was idle 10 days, *Answer, as before.*

4. A man had two silver cups of unequal weight, having one cover to both, which weighs 5oz.; now if the cover is put on the less cup it will be double the weight of the greater, and if put on the greater, it will be three times as heavy as the less; required the weight of each?

Ans. 3oz. less—and 4oz. greater.

5. There is a fish whose head is 9 inches long, his tail as long as his head and half his body, and his body as long as his head and tail both; what is its whole length? *Ans.* 6 *feet.*

6. A person being asked, in the afternoon, what o'clock it was, answered, that the time past from noon was equal to $\frac{2}{13}$ of the time to midnight; required the time? *Ans* 36*m. past one.*

7. A & B, settling accounts found, that if £6 were added to ⅔ of A's bill, and the same sum taken from ⅔ of B's, the *sum* would be ⅔ ~

remainder; and that the sum and remainder added together made £72 pounds; what was each man's bill?

Ans. A's £41 8*s.*, *and B's* £66 12*s.*

8. When first the marriage knot was tied
Between my wife and me,
My age did her's as far exceed,
As three times three does three;
But after ten and half ten years,
We man and wife had been,
Her age came up as near to mine
As eight is to sixteen.
Now, Tyro, skill'd in numbers, say,
What were our ages on the wedding day? *Ans.* 45 & 15.

THEORETICAL QUESTIONS. What is position? —— single position? —— the rule? —— double position? —— the rule? What is the better way of working questions that belong to this rule?

PERMUTATION AND COMBINATION.

227. PERMUTATION is finding how many *different ways* the order of things may be varied or changed.

1. Four persons agreed to board together so long as they could sit in different position at the table at dinner; pray, how long did they tarry?

If there had been TWO persons, *a* & *b*, they could sit in ($1 \times 2 =$) 2 positions; *a b* and *b a*; had there been THREE, they could sit in ($1 \times 2 \times 3 =$) 6 positions; for beginning with each one, we shall have TWO *positions* EACH *time*; thus, *a b c*, *a c b*; *b a c*, *b c a*; and *c a b*, *c b a*. In like manner, if there be FOUR, the different positions will be $1 \times 2 \times 3 \times 4 = 24$. *Ans.* 24 *days.*

228. Hence, to find the number of different changes or permutations of which any number of different things are capable,

RULE. Multiply together all the terms of the natural series of numbers, from 1 up to the given number, and the last product will be the answer.

2. Christ Church in Boston has 8 bells; how many changes may be rung on them? *Ans.* 40320.

3. How many variations will the nine digits admit of? *Ans.* 362880.

4. How many changes or variations may be made of the 26 letters of the alphabet? *Ans.* 6204484017332394393600000.

229. COMBINATION is finding how often a less number of things can be chosen from a greater.

We have seen, that any 3 things, *a*, *b*, *c*, all different from each other, admit of 6 variations; but if the things are all alike, as *a*, *a*, *a*, then the 6 variations will be reduced to 1, which may be expressed thus, $\frac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. Again, if but two things out of three are alike, as *a*, *a*, *c*: then the 6 variations will be reduced to three 3, *a a c*, *c a a*, *a c a*, which may be expressed thus, $\frac{2 \times 3}{1 \times 2} = 3$. And so on.

230. Hence, *To find the* COMBINATIONS of any given number of *things*, *all* different from each other, taking any given number at a time

RULE I. Multiply together the natural series, 1, 2, 3, &c. up to the number to be taken at a time.

II. Take a series of as many terms, decreasing by 1, from the number, out of which the choice is to be made, and find their continued product.

III. Divide this last product by the former, and the quotient will be the answer.

1. How many combinations may be made of 7 letters out of 12 ?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$ (the number to be taken at a time) $= 5040$.
$12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$ (same number from 12) $= 3991680$.
Then 5040)3991680(792, *Ans.*

2. A general was asked by his king what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men: what did it amount to? *Ans.* £23836022841 7*s.* $11\frac{65}{1134}$ *d.*

3. A butcher agreed with a farmer for a dozen of sheep, at $2 a head, which were to be picked out of 2 dozen; but being long in choosing, the farmer told him that if he would give him a cent for every different dozen which might be chosen out of the two dozen, he should have the whole, which was agreed to: pray what did the sheep cost him ?

Ans. $27041.56.

THEORETICAL QUESTIONS. What is permutation ? What the RULE ? What is combination ? —— the RULE ?

DUODECIMALS.

231. DUODECIMALS,* that is, TWELFTH PARTS, are chiefly used in measuring *surfaces* and *solids*.

232. In duodecimals, A FOOT is divided into TWELVE equal parts, called *inches* or *primes*, marked thus, (′); and each of these again into TWELVE other equal parts, called *seconds*, (″). In like manner, each second is divided into twelve equal parts, called thirds, (‴); each third also, into twelve equal parts, called fourths, (⁗); and so on.

In this way of dividing A FOOT, it is plain, that

1′ *inch* or *prime*, is	$\frac{1}{12}$	OF A FOOT.
1″ second is $\frac{1}{12}$ of $\frac{1}{12}$	$\frac{1}{144}$	... .
1‴ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$	$\frac{1}{1728}$	... .
1⁗ fourth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$	$\frac{1}{20736}$	... .

233. Duodecimals are *added* and *subtracted* the same as compound numbers, 12 OF A LESS denomination making 1 OF A GREATER, as in the following

TABLE.

12⁗ fourths	make	1‴ third.
12‴ thirds	,,	1″ second.
12″ seconds	,,	1 inch or prime.
12 inches or primes		1 FOOT.

234. The marks, ′ ″ ‴ &c., are called the INDICES.

* Derived from the Latin *duodecim*, which signifies twelve.

MULTIPLICATION OF DUODECIMALS.

235. IN SUPERFICIES *the* LENGTH *multiplied by the* BREADTH *gives the* SUPERFICIAL CONTENTS.

1. How many square feet in a board 15ft. 9′in. long, and 1ft. 9′in. wide?

15	9′		
1	5′		
6	6′	9″	
15	9′		
22	3′	9″	

9′ inches or primes $\times$ 5′ $= \frac{9}{12} \times \frac{5}{1} = \frac{45}{144}$ or 45″; 45″ $=$ 3′ 9″. We set down 9″, and reserve the 3 to carry ..... 15ft $\times$ 5′ $= \frac{15}{1} \times \frac{5}{12} = \frac{75}{12}$ or 75′, and 3′ we had to carry, gives 78′ $=$ 6 ft. 6′ inches. We set down the 6′, and carry the 6ft. to the left. Then, multiplying by the 1ft., and adding the two products together, we obtain 22ft. 3′ 9″, *Answer.*

Hence we see, (AND LET IT BE REMEMBERED,) that

236. The product of any two denominations will always be of THAT denomination denoted by the SUM OF THEIR INDICES.

2. How many cubic feet in a block 13ft. 9′ long, 11 ft. 10′ wide, and 9ft. 7′ thick?

NOTE. It will generally be found most convenient, to multiply FIRST by the *feet* of the multiplier, then by the *inches*, and so on.

Length.	13	9′		
Breadth.	11	10′		
	151	3′		
	11	5′	6″	
	162	9′	6″	
Thickness.	9	7′		
	1464	4′	6″	
	94	10′	11″	6‴
Answer,	1559	2′	5″	6‴.

236 *a.* In SOLIDS the LENGTH multiplied by the BREADTH, and that product by the THICKNESS, gives the SOLID CONTENTS.

From these examples we derive the following

237. RULE. Write down the denominations as compound numbers, and in multiplying, REMEMBER, to make the product of any two denominations TO BE OF THAT denomination denoted by the SUM OF THEIR INDICES.

EXAMPLES IN DUODECIMALS.

1. How many square feet in a board 17ft. 7′ long, 1ft. 5′ wide?
Ans. 24*ft.* 10′ 11″.
2. How many solid feet in a stick of timber 12ft. 10′ long, 1ft. 7′ wide, and 1ft. 9′ thick?
Ans. 35*ft.* 6′ 8″ 6‴.
3. How many feet in a load 6ft. 7′ long, 3ft. 5′ high, and 3ft. 9′ wide?
Ans. 82*ft.* 5′ 8″ 4‴.
4. How much wood in a load 11ft. 4in. long, 5ft. 9in. wide, and 3ft. 11in. high?
Ans. 225ft. 2′ 10″.
5. How many solid feet in a block 15ft. 8′ long, 1ft. 5′ wide, and 1ft. 4′ thick?
Ans. 29*ft.* 7′ 1″ 4‴.
6. What is the product of 371ft. 2′ 6″ multiplied by 181ft. 1′ 9″?
Ans. 67242*ft.* 10′ 1″ 4‴ 6‴.
7. How many cords, &c., in a pile 176ft. in length, 3ft. 9′ wide, and 4ft. 3′ high?
Ans. 21 C. 117*ft.*
8. What is the price of a marble slab, whose length is 5ft 7′, and 1ft. 10′, at $1 per foot?
Ans. $10.23.
9. There is a house with three tiers of windows, 3 in a tier, the height of the first tier is 7ft. 10′, of the second 6ft. 9′, of the third 5ft. 4′, and the

breadth of each is 3ft. 11′; what will the glazing come to at 14d. per foot?
Ans. £13 11*s.* 10½*d.*

10. If a house measures within the walls 52ft. 8′ in length, and 30ft. 6′ in breadth, and the roof be of a true pitch, or the rafters ¾ of the breadth of the building, what will the roofing come to, at 10s. 6d. per square?
Ans. £12 12*s.* 11¾*d.*

THEORETICAL QUESTIONS.

What are duodecimals? In what are they chiefly used? In duodecimals, how is a foot divided? How are duodecimals added or subtracted? What are the marks that distinguish the denominations, called? How do you find superficial contents? —— solid contents? What is the RULE FOR DUODECIMALS?

INVOLUTION,

OR THE

RAISING OF POWERS.

238. INVOLUTION is multiplying any number into itself a certain number of times.

239. The PRODUCTS obtained are called POWERS.

240. The NUMBER ITSELF is called the ROOT or *first* power.

241. If the ROOT or *first power* be multiplied into itself, the product is called the SQUARE or *second* power; if the square be multiplied by the first power, the product is called the CUBE or *third* power, &c.

Thus, of the number 5,

5 is the root, or 1st power.
5 × 5 = 25 is the square, or 2d power = 5^2.
5 × 5 × 5 = 125 is the cube, or 3d power = 5^3.
5 × 5 × 5 × 5 = 625 is the biquadrate, or 4th power = 5^4.

242. The *little figure* denoting the power is called the INDEX OR EXPONENT.

EXAMPLES.

1. What is the square or 2d power of 7? *Ans.* 49.
2. What is the square of 9? —— the cube of 9? —— the 4th power of 9? *Ans.* 81, 729, 6561.
3. What is the square of 30? *Ans.* 900.
4. " " " " of 400? *Ans.* 160000.
5. " " " " of 8000? *Ans.* 64000000.
6. What is the cube of 1? —— of 2? —— of 3? —— of 4? —— of 6? —— of 7? *Ans.* 1, 8, 27, 64, 216, 343.
7. What is the square of $\frac{2}{3}$? —— $\frac{3}{4}$? —— $\frac{4}{5}$? —— $\frac{5}{6}$? *Ans.* $\frac{4}{9}$, $\frac{9}{16}$, $\frac{16}{25}$, $\frac{25}{36}$.

243. It is evident, that a fraction is involved *by involving the* NUMERATOR AND DENOMINATOR. (Art. 121.)

8. What is the square of $\frac{1}{4}$? —— the cube? —— the 4th power?
Ans. $\frac{1}{16}$, $\frac{1}{64}$, $\frac{1}{256}$.

9. What is the square, cube, and 4th power of .5?
Ans. .25, .125, & .0625.

10. What is the square, cube, and 4th power of 1.5?
Ans. 2.25, 3.375, & 5.0625.

11. What is the 6th power of $2\frac{1}{4}$?

244. A mixed number is best involved *by first reducing it to an* IMPROPER FRACTION. $2\frac{1}{3} = \frac{7}{3}$, and the 6th power of $\frac{7}{3}$ is $\frac{117649}{729} = 161\frac{280}{729}$.

12. Involve $2\frac{1}{4}$ to the 6th power. *Ans.* $\frac{531441}{4096} = 129\frac{3217}{4096}$.

13. Involve 2^6, that is to say, involve 2 to the 6th power. *Ans.* 64.

14. Involve 3^7? —— 4^6? —— 6^8? —— 7^9? —— 8^{10}? —— 10^8? —— 9^{11}? —— 13^{12}? —— 12^{13}?
Ans. to last, 106993205379072.

15. What is the square of 99? *Ans.* 9878.

The powers of the 9 digits, from the 1st power to the 5th, may be seen in the following

TABLE.

Roots -	or 1st powers	1	2	3	4	5	6	7	8	9
Squares	or 2d powers	1	4	9	16	25	36	49	64	81
Cubes -	or 3d powers	1	8	27	64	125	216	343	512	729
Biquadrates	or 4th powers	1	16	81	256	625	1296	2401	4096	6561
Surdsolids	or 5th powers	1	32	243	1024	3125	7776	16807	32768	59049

EVOLUTION,

OR THE

EXTRACTION OF ROOTS.

245. EVOLUTION is finding the ROOT of any given power or number.

246. The *root* we have seen, is that number, which being multiplied into itself continually, will produce the given power.

247. The SQUARE ROOT of any given number is a number, which being multiplied into itself, will produce that number;

And the CUBE ROOT, is a number which being cubed, or involved to the 3d power, will produce the *same* given number.

The *square root* of 144 is 12, because $12^2 = 144$.
The *cube root* of 512 is 8, because ... $8^3 = 512$, &c.

248. This character $\surd$ placed before any number, expresses the square root of that number.

Thus $\surd$ 25 expresses the square root of 25.

249. The same character is made to express any other root, by placing the index of the root above it.

Thus $\sqrt[3]{}$ 27 expresses the cube root of 27.

And $\sqrt[4]{}$ 625 expresses the 4th root of 625, &c.

250. It appears from the preceding table of powers, that the *square* of any root can *have but* TWICE *as many* figures as the *root itself*.*

Hence we may always tell how many figures there will be in the SQUARE ROOT of any number, *by pointing it off from unit's place, into* PERIODS of TWO FIGURES *each*.

251. It also appears, that the *cube* of any root *can have, at most, but* THREE TIMES *as many figures* as *the root itself*. †

Hence we may ascertain how many figures there will be in the CUBE ROOT of any number, *by pointing it off from unit's place, into periods of* THREE FIGURES *each*.

252. Although there is no number which will not produce a perfect power, yet there are numbers of which *exact roots* can never be obtained.‡ Numbers, whose exact roots cannot be obtained, are called SURD NUMBERS, and those whose roots can be thus obtained, are called RATIONAL NUMBERS.

EXTRACTION OF THE SQUARE ROOT.

253. To EXTRACT THE SQUARE ROOT is to find a number, which being multiplied into itself, will produce the GIVEN number.

Required the side of a square, that shall contain 625 square feet.

The contents of a square, as we have seen, (page 49,) is found by *multiplying the length of one side* INTO ITSELF. Hence, if we have the contents of any square given, we may always find the *side* by *extracting the* SQUARE ROOT.

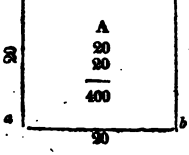
This is done by a SORT OF TRIAL.

'1st. We must ascertain how many figures there will be in the root. This we can do, *by pointing off the number from units, into periods of* TWO FIGURES *each*; which, in this case, being done, we find that the root will consist of two figures, a *ten* and a *unit*.

OPERATION.

```
. .
625 (2
4
---
225
```

Fig. I.



2d. We seek for the first figure, that is, for the TENS of the root, and it is plain, that we must extract it from the left hand period 6 (*hundred*). The greatest square in 6 (*hundred*) we find, by trial, to be 4 (*hundred*), the root of which is 2 (*tens* = 20,) we therefore set 2 (*tens*) in the root. The *root*, it will be recollected, is *one side* of a square. Let us, then, form a square, (A, Fig. I,) each side of which shall be supposed 2 tens, = 20ft., expressed by the root now obtained. The contents of this square is $20 \times 20 = 400$ ft., now disposed of, and which, consequently, is to be deducted from the whole number of feet, (625,) leaving 225ft. This deduction is most readily made by subtracting the square 4 (*hundred*), or the square of 2, (the figure in the root already found,) from the period 6 (*hundred*), and bringing down the next period by the side of the remainder, making 225 as before. 3d. The square A is now to be enlarged by the addition of the 225 remaining feet; and, in order that

* Because the square of 9, the *highest digit* contains but *two* figures.

† Because the cube of 9, contains but THREE figures.

‡ By the *help of decimals*, however, we can approximate towards the [illegible] *to any degree of exactness* required.

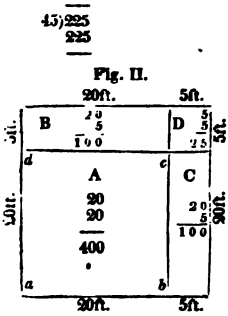
the figure may retain its *square form*, it is evident, the addition must be made on *two* sides. Now, if the 225 feet be divided by the *length* of the *two* sides, (20 + 20 = 40,) the quotient will be the *breadth* of this new addition of 225 feet to the sides c d and b c of the square A.

But our root already found, 2 tens, is the length of *one* side of the figure A; we therefore take *double* this root, = 4 tens for a DIVISOR.

OPERATION—CONTINUED.

625(25
4
---
45)225
225

Fig. II.



The divisor, 4, (*tens,*) is in reality 40, and we are to seek how many times 40 is contained in 225, or, which is the same thing, we may seek how many times 4 (tens,) is contained in 22 (tens,) rejecting the right hand figure of the dividend, because we have rejected the cipher in the divisor. We find our quotient, *that is*, the breadth of the addition, to be 5 feet; but if we look at Fig. II. we shall perceive that this addition of 5 feet to the *two* sides does not complete the square; for there is still wanting, in the corner D, a small square, each side of which is equal to this last quotient, 5; we must, therefore, add this quotient, 5, to the divisor, 40, that is, place it to the right hand of the 4 (*tens,*) making it 45; and then the whole divisor, 45, multiplied by the quotient, 5, will give the contents of the whole addition around the sides of the figure A, which, in this case, being 225 feet, the same as our dividend, we have no remainder, and the work is done. Consequently Fig. II. represents a square, one side of which is 25 feet, *the answer*.

If we would prove the operation, we may do it by adding together the several parts of the figure, thus:

The square A contains 400 feet
The figure B - - - - - 100 .....
- - - - - - C - - - - - 100 .....
- - - - - - D - - - - - 25 .....

Whole Fig. II. - - - - 625, Proof.

Or, by involution, thus:
$25 \times 25 = 625$, as before.

254. From the example and illustration now given, we derive the following

RULE

FOR EXTRACTING THE SQUARE ROOT.

I. POINT OFF the given number, from unit's place, into periods of TWO FIGURES each.

II. Find the greatest SQUARE NUMBER in the left hand period, place its root in the quotient; subtract the square number from the left hand period, and to the remainder bring down the next period for a DIVIDEND.

III. DOUBLE the root already found for a DIVISOR; seek how many times the divisor is contained in the dividend, (rejecting the right hand figures,) and place the result in the quotient, and also at the right hand of the divisor; multiply the divisor thus augmented, by the last quotient figure, and subtract the product from the dividend; to the remainder bring down the next period, and proceed as before.

NOTE. As the value of figures, whether *integers* or *decimals*, is determined by their distance of units, so we must always begin at unit's place to

point off. Mixed decimals must, therefore, be pointed off *both* ways from units, and if there be a deficiency in any period of decimals, it must be supplied by a cipher.

1. What is the square root of 207936?

$$\begin{array}{l} \dot{2}0\dot{7}9\dot{3}\dot{6}(456, \textit{Ans.} \\ 16 \\ \hline 85)\ 479 \\ \quad 425 \\ \hline 906)\ 5436 \\ \quad 5436 \\ \hline \end{array}$$

2. What is the square root of 22071204? *Ans.* 4698.
3. What is the square root of 94249?

$$\begin{array}{l} \dot{9}4\dot{2}4\dot{9}(307, \textit{Ans.} \\ 9 \\ \hline 607)\ 4249 \\ \quad 4249 \\ \hline \end{array}$$

4. What is the square root of 10342656? *Ans.* 3216.
5. What is the square root of 36372961? *Ans.* 6031.
6. What is the square root of 234.09? *Ans.* 15.3.
7. What is the square root of .00003169? *Ans.* .00563.
8. What is the square root of 964.5192360241? *Ans.* 31.05671.
9. What is the square root of 160?

$$\begin{array}{l} \dot{1}6\dot{0}(12.64+, \textit{Ans.} \\ 1 \\ \hline 22)\ 60 \\ \quad 44 \\ \hline 246)1600 \\ \quad 1476 \\ \hline 2524)\ 12400 \\ \quad 10096 \\ \hline \end{array}$$

255. Here, as there was a remainder, after having brought down all the figures, we continued the operation to decimals, *by annexing two ciphers, each time, for a new period. In this way, the root may be carried to any degree of exactness required.*

10. What is the square root of 7? —— of 10?
Ans. 2.645751+ & 3.162277+.
11. What is the square root of .00032754? *Ans.* .018109+.
12. What is the square root of $\frac{4}{9}$?

256. We have seen, (Art. 243,) that a fraction is involved by involving the *numerator* and *denominator*. Hence, it is evident that the ROOT of a fraction may be obtained *by extracting the root of the* NUMERATOR *and of the* DENOMINATOR. *Ans. to the above,* $\frac{2}{3}$.

13. What is the square root of $\frac{16}{49}$? *Ans.* $\frac{4}{7}$.
14. What is the square root of $\frac{25}{100}$? *Ans.* $\frac{5}{10}$.
15. What is the square root of $2\frac{1}{4}$? $2\frac{1}{4} = \frac{9}{4}$ *and* $\sqrt{\frac{9}{4}} = \frac{3}{2} = 1\frac{1}{2}$, *Ans.*

Or, $2\frac{1}{4} = 2.25$, & $\sqrt{2.25} = 1.5$, *as before.*

257. When the numerator and denominator are not *exact powers*

the fraction may be reduced to a decimal, and the approximate root found as above.

16. What is the square root of $\frac{4}{5}$? *Ans.* .89442719

THEORETICAL QUESTIONS.

What is involution? What are powers? —— the 1st power? —— the 2nd power? —— the 3d power? What is the index, or exponent? How is a fraction involved? —— a mixed number? What is evolution? What is a root? —— the *square root*? —— the cube root? What does the character $\surd$, or an inverted *r*, express? How is the same character made to express any other root? Of how many figures at most, can the square of any root consist? Why? How, then, can we tell how many figures there will be in the *square root* of any number? Of how many figures can the cube of any root consist? How, then, can the number of figures in the *cube* root of any number be ascertained? Can the exact root of all numbers be found? What are surd numbers? —— rational numbers? —— What is it to extract the square root? In the operation, why do we double the root for a divisor? Why do we, in dividing, reject the right hand figure of the dividend? Why do we place the quotient figure to the right hand of the divisor?

EXAMPLES FOR PRACTICE.

1. If 1369 fruit trees be planted in a square orchard, how many must be in a row each way? *Ans.* 37.

2. If a square field contain 2025 square rods, how many rods will it measure on each side? *Ans.* 45.

3. There is an army of 4096 men; how many must be placed in rank and file to form them into a square? *Ans.* 64.

4. There is a circle, whose *area*, or superficial contents, is 5625 feet; what will be the length of one side of an equal square?

$\surd 5625 = 75$, *Ans.*

5. A man has two fields, one containing 40 acres, and the other 50; B offers him, in exchange, a square field containing the same number of acres; how many rods must each side of this field measure?

Ans. 120 *rods.*

6. Suppose I have an eliptical fish pond, containing 9A. 2R. 15 poles, and would have a square one of the same superficies, what will be the length of each side? *Ans.* 215.484918 *yards.*

7. There is a square field, which measures 20 rods on each side; what will be the side of a square, which contains 4 times as much?

$\surd 20 \times 20 \times 4 = 40$ *rods*, *Ans.*

8. If the side of a square be 3 feet, what will be the side of one 3 times as large? —— 9 times as large? —— 25 times as large? —— 36 times as large? *Ans. to last*, 18 *feet.*

9. It is required to lay out 338 rods of land in the form of a parallelogram, so that it shall be twice as many rods in length as it is in breadth?

If the field be divided in the middle, it will form *two equal* squares; hence $338 \div 2 = 169$, then $\surd 169 = 13$ rods, *the width*; and $13 \times 2 = 26$ rods, *the length.*

10. I would set out at equal distances 1444 apple trees, so that my orchard may be 4 times as long as it is wide; how many rows of trees must I have, and how many trees in each row?

Ans. 19 *rows, and* 76 *trees in each row.*

11. There is an oblong piece of land, containing 192 square rods, of which the breadth is $\frac{3}{4}$ as much as the length; required its length and

ves from the length, it will form a square, of which the length on each

side is equal to the width of the oblong: hence the area of this square will be $\frac{1}{4}$ less than that of the oblong; $192 - \frac{192}{4} = 144$, and $\sqrt{144} = 12$, the width; then $12 + \frac{12}{4} = 16$, the length.

12. Suppose 450 men be drawn up in oblong battle, so that the number in rank be $12\frac{1}{2}$ times as many as in file; required the number in rank and file? *Ans.* 6 *and* 75.

13. There is a parallelogram, containing 2744 square rods, the breadth of which is $\frac{1}{4}$ as much as the length; required its dimensions. *Ans.* 56 *by* 49.

14. There is a circle whose diameter is 3 inches; what is the diameter of one 4 times as large?

258. The AREAS of circles are in proportion to the SQUARES of their DIAMETERS.

Hence, if the diameter of any circle be given, to find the diameter of one, 2, 3, $\frac{1}{2}$, $\frac{1}{3}$, &c., times as large,

RULE. Multiply the square of the diameter by the GIVEN RATIO; and the square root of the product will be the answer.

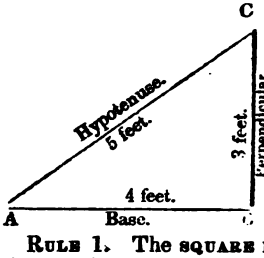
Applying the rule, ... $\sqrt{3 \times 3 \times 4} = 6$, *Ans, to the above.*

15. If the diameter of a circle be 5 feet, what will be the diameter of one 4 times as large? —— 9 times as large? —— 16 times as large? —— 25 times as large? *Ans. to last,* 25 *feet.*

16. If the diameter of a circle be 12 feet, what will be the diameter of one $\frac{1}{4}$ as large? *Ans.* 6 *feet.*

17. There are two circular ponds in a gentleman's pleasure ground; the diameter of the less is 48 feet, and the greater is 3 times as large; what is its diameter? *Ans.* 83.13+.

259. 18. A carpenter has a large *wooden square;* one part of it is 4 feet, and the other part 3; what length of cord will reach from one end to the other?



DEFINITION 1. A figure having *three sides,* is called a TRIANGLE.

2. If one of the corners of a triangle be a *square corner,* as at B, it is called a RIGHT ANGLED TRIANGLE.

In any right angled triangle the *square of the longest side,* or HYPOTENUSE, is equal to the SUM OF THE SQUARES *of the two other sides;* consequently the DIFFERENCE OF THE SQUARES of the hypotenuse and EITHER of the other sides is the square of the remaining side. Hence,

RULE 1. The SQUARE ROOT of the SUM OF THE SQUARES of the two shortest sides, is the length of the HYPOTENUSE.

RULE 2. The SQUARE ROOT of the DIFFERENCE OF THE SQUARES of the HYPOTENUSE and EITHER of the other sides, is the length of the REMAINING side.

Applying the rule to example 18, $\sqrt{4^2 + 3^2} = 5$ feet, *Ans.*

19. If the two legs, that is, the two shortest sides, of a triangle be 12 inches each, what is the length of the hypotenuse?

Ans. 16.97 +. But the carpenters use 17 for their *square rule.*

20. The wall of a town is 25 feet high, and is surrounded by a moat of 30 feet in breadth; I desire to know the length of a ladder that will reach from the outside of the moat to the top of the wall. *Ans.* 39.05 *feet.*

EXTRACTION OF THE CUBE ROOT.

260. A CUBE is a solid body having *six equal sides*, and each of the sides an EXACT SQUARE.

261. *The* ROOT *of a* CUBE *is, therefore, the measure in* LENGTH *of* ONE *of its* SIDES.

For the length, breadth, and thickness, of such a body are ALL EQUAL.

Consequently, the length of *one side* of a cube, raised to the 3d power, gives the solid contents.

262. EXTRACTING THE CUBE ROOT of any number is therefore *finding the length of* ONE SIDE *of a cube whose* SOLIDITY *is equal to* THAT NUMBER.

1. Required the solid contents of a cubic block, one side of which measures 5 feet. $5^3 = 125$ feet, *Ans.*

2. Required the length of one side of a cubic block, that shall contain 125 solid feet. $\sqrt[3]{125} = 5$ feet, *Ans.*

3. A man has 13824 feet of timber, in separate blocks of 1 cubic foot each, and wishes to put them up in a cubic pile, what will be the length of one side of such a pile?

The answer to this question is evidently found by *extracting the cube root* of the whole number of feet, (13824.)

To do which we must

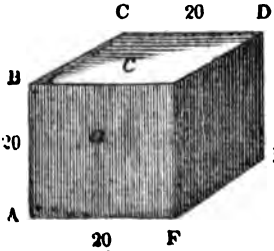
1st. Ascertain the number of figures, of which the root will consist. This we do by *pointing off the given number into periods of* THREE FIGURES *each*. Thus we find, that the root will consist of two figures, a ten and a *unit*.

OPERATION.

13824(2
8

5824

FIG. I.



20
20

400
20

9000 *ft. contents of the above* FIG.

'Let us, then, seek for the first figure, or tens of the root, which must evidently be extracted from the left hand period, 13 (thousand). The greatest cube in 13 (thousand) we find by trial to be 8 (thousand) the root of which is 2 (tens;) we therefore place 2 (tens) in the root. The root, it will be recollected, is one side of a cube. Let us, then, form a cube, (Fig. I,) each side of which shall be supposed 20 feet, expressed by the root now obtained. The contents of this cube are $20 \times 20 \times 20 = 8000$ solid feet, which are now disposed of, and which, consequently, are to be deducted from the whole number of feet, 13824. 8000 feet taken from 13824 leaves 5824 feet. This deduction is most readily performed by subtracting the cubic number, 8, or the cube of 2, (the figure of the root already found,) from the period 13 (thousand) and bringing down the next period by the side of the remainder, making 5824, as before.

2nd. The cubic pile, A D, is now to be enlarged by the addition of 5824 solid feet, and in order to preserve the cubic form of the pile, the addition must be made on one half of its sides, that is, on 3 sides, *a*, *b*, and *c*. Now, if the 5824 solid feet be divided by the square contents of these 3 equal sides, that is, by 3 times, $(20 \times 20 = 400) = 1200$, the quotient will be the thickness of the addition to be made to

each of the sides *a*, *b*, and *c*. But the root, 2, (tens,) already found is the length of one of these sides; we therefore square the root 2 (tens) = 20 × 20 = 400, for the *square contents* of *one* side, and multiply the product by 3, the *number* of sides; 400 × 3 = 1200: or, which is the same, in effect, and more convenient in practice, we may square the 2 (tens) and multiply the product by 300, thus, 2 × 2 = 4, and 4 × 300 = 1200, for the divisor, as before.

OPERATION—CONTINUED.

13824(24 *Root.*
8

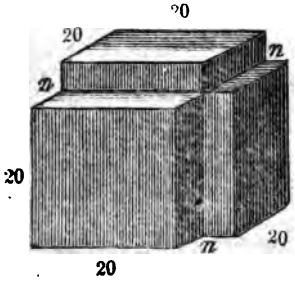
Divisor, 1200)5824 *Dividend.*

4800
960
64

5824

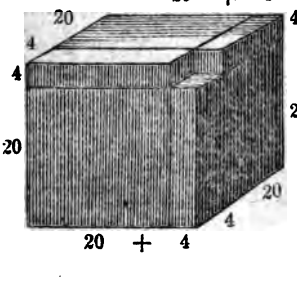
0000

FIG. II.



The divisor, 1200, is contained in the dividend 4 times; consequently, 4 feet is the thickness of the addition made to each of the three sides, *a*, *b*, *c*. And 4 × 1200 = 4800, is the solid feet contained in these additions; but, if we look at Fig. II, we shall perceive that this addition to the 3 sides, does not complete the cube; for there are deficiencies in the 3 corners *n*, *n*, *n*. Now the *length of each side*, that is, 2 (tens) = 20, and their *width* and *thickness*, are each equal to the *last quotient* figure (4); their contents, therefore, or the number of feet required to *fill* these deficiencies, will be found by multiplying the square of the last quotient figure, (4^2) = 16, by the length of *all* the deficiencies, that is, by 3 times the length of each side, which is expressed by the former quotient figure 2 (tens,) 3 times (2 tens) is 6 (tens) = 60; or, which is the same, in effect, and more convenient in practice, we may multiply the quotient figure 2 (tens) by 30, thus, 2 × 30 = 60, as before, then 60 × 16 = 960 contents of the three deficiencies *n*, *n*, *n*.

FIG. III.



Looking at Fig. III., we perceive there is still a deficiency in the corner where the last blocks meet. This deficiency is a cube, each side of which is equal to the last quotient figure 4. The cube of 4, therefore, (4 × 4 × 4 = 64,) will be the solid contents of this corner, which in Fig. IV, is seen filled.

Now the sum of these several additions, viz., 4800 + 960 + 64 = 5824, will make the subtrahend, which, subtracted from the dividend, leaves no remainder, and the work is done.

FIG. IV.

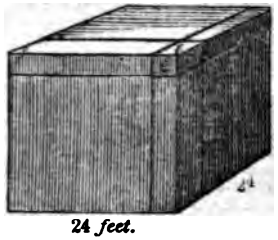


Fig. IV. shows the pile which 13824 solid blocks of one foot each would make, when laid together, and the root, 24, shows the length of one side of the pile. The ROOT may be obtained by cubing the side now found, ($24^3 = 13824$); or by adding together the contents of all the several parts; thus,

Feet.
8000 = contents of Fig. I.
4800 = addition to the sides *a*, *b*, and *c*, Fig. I.
960 = addition to fill the deficiencies *x*, *x*, *x*, Fig. II.
64 = addition to fill the corner *e*, *e*, *e*, Fig. IV.

13824 = contents of the whole pile, Fig. IV., which is 24 feet on each side.

263. From the foregoing example and illustration we have the following

RULE

FOR EXTRACTING THE CUBE ROOT.

I. POINT OFF the given number, from unit's place, into PERIODS OF THREE FIGURES each.

II. FIND the greatest cube in the left hand period, and put its ROOT in the quotient.

III. SUBTRACT the root thus found from the said period, and to the remainder bring down the next period, and call the sum which it makes the DIVIDEND.

IV. MULTIPLY the square of the quotient by 300, and call it the DIVISOR.

V. SEEK how often the divisor may be had in the dividend, and place the result in the quotient; multiply the divisor by this quotient figure, and write the product under the dividend.

VI. MULTIPLY the square of this quotient figure by the former FIGURE, or FIGURES, of the root, and this product by 30, and place the product under the last; under ALL write the cube of the last quotient figure, and call their sum the SUBTRAHEND.

VII. SUBTRACT the subtrahend from the dividend, and proceed as before.

NOTE 1. If the divisor cannot be had in the dividend, put a cipher in the root, bring down the next period for a new dividend, and proceed as before directed.

2. If necessary, the root may be carried to decimals, by annexing *three* ciphers to each subtrahend, and continuing the operation.

EXAMPLES.

1. What is the cube root of 1740992427?

$$\begin{array}{rl} & \overline{1}740992427\,(1203,\ \textit{Ans.}\\ & 1\\ \hline 1^2\times 300=300) & 740 \text{ Dividend.}\\ & 600\\ \hline 2^2\times 1\times 30=120 & \\ 2^2=\ 8 & \\ \hline & 728 \text{ Subtrahend.}\\ \hline 12^2\times 300=43200) & 12992 \text{ Dividend.}\\ \hline 120^2\times 300=4320000) & 12992427 \text{ Dividend.}\\ \hline & 12960000\\ 3^2\times 120\times 30= & 32400\\ 3^3= & 27\\ \hline & 12992427\\ \hline \end{array}$$

2. What is the cube root of 941192? *Ans.* 98.
3. What is the cube root of 21024576? *Ans.* 276.
4. What is the cube root of 1392155106928543? *Ans.* 240607.
5. What is the cube root of 12.977875? *Ans.* 2.35.
6. What is the cube root of 36155.027576? *Ans.* 33.06+
7. What is the cube root of .001906624? *Ans.* .154.
8. What is the cube root of .000729? *Ans.* .09.
9. What is the cube root of $\frac{27}{125}$? *Ans.* $\frac{3}{5}$

NOTE. For principles, see Arts. 256 and 257.

10. What is the cube root of $\frac{216}{343}$? *Ans.* $\frac{6}{7}$
11. What is the cube root of $1\frac{37}{1728}$? *Ans.* $1\frac{1}{12}$
12. What is the cube root of $\frac{1}{500}$? *Ans.* .125+
13. What is the cube root of $\frac{1}{5000}$? *Ans.* .058+

THEORETICAL QUESTIONS.

What is a cube? What is the root of a cube? What is it to extract the cube root? Why is the square of the quotient multiplied by 300? Why, in finding the subtrahend, do we multiply the square of the last quotient figure by 30 times the former figure or figures of the root? Why do we cube the quotient figure? How may we prove the operation?

EXAMPLES FOR PRACTICE.

1. How many solid feet are there in a cubical box, one side of which measures 2 feet? *Ans.* 8 *feet.*
2. How many cubic feet in one 8 times as large? and what would be the length of one side? *Ans.* 64, *and one side would be* 4 *feet.*
3. What is the side of a cubical pile, equal to one 576 feet long, 432 feet wide, and 96 feet high? *Ans.* 288 *feet.*

4. There is a cubical box, measuring 1 foot each way; what is the side of a box 8 times as large? —— 27 times? —— 64 times?

Ans. 2, 3, *and* 4 *feet.*

264. Hence, it appears, that the sides of cubes are as the cube roots of their solid contents, and consequently, their CONTENTS are as the CUBES of their SIDES. The same is also true of the SIMILAR SIDES, or of the DIAMETERS of all solid figures of SIMILAR FORMS.*

5. There is a cubical box, one side of which is 5 feet; what would be the side of one containing 27 times as much? —— 64 times as much? —— 125 times as much?

Ans. 15, 20, *and* 25.

6. If a ball, weighing 4 pounds, be 3 inches in diameter, what will be the diameter of a ball of the same metal, weighing 32 pounds?

$$4 : 32 :: 3^3 : 216, \text{ and } \sqrt[3]{216} = 6 \textit{ in.}$$ *Ans.*

7. If a ball, 6 inches in diameter, weigh 32 pounds, what will be the weight of a ball 3 inches in diameter?

Ans. 4*lbs.*

8. If a cube of silver, whose side is 4 inches, be worth $125, what will be the side of one worth 4 times as much?

Ans. 6.349*in.*+

9. If a globe of silver, of 3 inches diameter, be worth $100, what will be the value of another globe 1 foot in diameter?

Ans. $6400.

10. There are two globes; one of them is 1 foot in diameter, and the other 30 feet: how many of the smaller globes will it take to make one of the larger?

Ans. 27000.

11. There is a cubical vessel, whose side is 2 feet; I demand the side of a vessel which shall contain 3 times as much.

Ans. 34$\frac{1}{2}$*in.*

12. If the diameter of the sun be 112 times that of the earth, how many globes like the earth would it take, to make one as large as the sun?

Ans. 1404928.

13. If the planet saturn is 1000 times larger than the earth, and the earth is 7900 miles in diameter, what is the diameter of saturn?

Ans. 79000 *miles.*

14. If the British national debt be 13 hundred millions of pounds sterling, what would be the diameter of a globe of gold of equal value, allowing a cubic inch of the metal to be worth £53 2*s.* 8*d.*? †

Ans. 360.199944492 *inches.*

ARITHMETICAL PROGRESSION.

265. ARITHMETICAL PROGRESSION is any rank or series of numbers, *more than two,* INCREASING OR DECREASING by a *constant difference.*

266. If the numbers *increase,* they are said to form an ASCENDING SERIES; but if they *decrease,* they form a DESCENDING SERIES.

Thus, { 3, 5, 7, 9, 11, 13, &c. is an *ascending* series.
{ 13, 11, 9, 7, 5, 3, &c. ... a *descending* series.

267. The numbers which form the series are called the

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* How are the sides of cubes proportional? —— their CONTENTS?

† The solidity of a globe divided by .5236 gives the cube of the diameter.

TERMS of the series; of which the *first* and *last*, are called the EXTREMES, and the other terms the MEANS.

268. There are FIVE THINGS in arithmetical progression, any *three* of which being given, the other two may be found.

1st. The FIRST term.
2nd. The LAST term.
3d. The NUMBER of terms.
4th. The COMMON DIFFERENCE.
5th. The SUM OF ALL THE TERMS.

1. Bought 10yds. of cloth, giving 3 cents for the first yard, 5 for the second, 7 for the third, and so on; what did the last yard cost me?

The *common difference*, 2, being added to every term except

1st. term 2d. 3d. 4th. 5th. 6th. 7th. 8th. 9th. 10th.
$3+2=5+2=7+2=9+2=11+2=13+2=15+2=17+2=19+2=21.$

the last, it is plain, the *last* yard will be $9 \times 2 = 18$ cents *more* than the *first*. Therefore $3 + 18 = 21$ cents, *Ans.*

Hence, when the first term, the common difference, and the number of terms, are given, to find the last term,

269. RULE. Multiply the number of terms LESS ONE, by the common difference, and to the product add the first term.

2. A person travelling into the country, went 3 miles the first day, 8 the second, and so on, for 12 days; how far did he travel the last day?

Ans. 58 *miles.*

3. A man had 8 sons, whose ages differed alike, the youngest was 4 years old, and the common difference 4; what was the age of the eldest?

Ans. 32 *years.*

4. What will be the amount of $1 for 50 years, at 6 per cent.?

270. It is plain, the yearly amounts, at simple interest, form an arithmetical progression, of which the *principal* is the *first term*, the *last amount* the *last term*, the yearly interest the *common difference*, and the number of *years* 1 less than the *number of terms.* *Ans.* $4.

5. A gentleman bought 100yds. of Osnaburgh, and by agreement was to give what the last yard would come to, reckoning 2s. for the first, 5 for the second, 8 for the third, and so on; what did it cost him?

Ans. £14 19*s.*

---

6. Bought 10yds. of cloth in arithmetical progression, giving for the first yard 3 cents, and for the last 21; what was the common difference?

This being the reverse of example 1st, we proceed with it inversely; thus, $21 - 3 = 18$, and $18 \div 9 = 2$, *Ans.*

Hence, *when the extremes and number of terms are given, to find the common difference,*

271. RULE. Divide the difference of the extremes by the number of terms LESS ONE, and the quotient will be the answer.

7. If the extremes be 7 and 315, and number of terms 78; what is the common difference? *Ans.* 4.

8. A man had 8 sons in arithmetical progression, the youngest was 4 years old, and the eldest 32; what was the common difference of their ages? *Ans.* 4 *years.*

9. If A puts out $1, at simple interest, for 50 years, and receives at the end of the time $3; what is the *common difference*, or *rate per cent.*?

Ans. 8 %

10. A man bought 10yds. of cloth in arithmetical progression, giving for the first yard 3 cents, and for the last 21; what was the average price per yard, and what was the amount of the whole?

As the price of each succeeding yard increases by a *constant difference*, it is plain that the *average* price will be as much *less* than the price of the last yard as it is *greater*, than the price of the *first*; consequently one half the sum of the *first* and *last* price will be the *average price*.

$3 + 21 = 24$cts., & $\frac{1}{2}$ of $24 = 12$cts. average price; } *Ans.*
Consequently, 12cts. $\times$ 10 = 120cts., whole cost, }

Hence, *when the extremes and number of terms are given, to find the sum of all the terms,*

272. RULE. Multiply $\frac{1}{2}$ the sum of the extremes by the number of terms, and the product will be the answer.

11. If the extremes be 7 and 315, and the number of terms 78; what is the sum of the series? *Ans.* 12558.

12. How many times does the hammer of a clock strike in 12 hours? *Ans.* 78.

13. What is the sum of the first 1000 numbers? *Ans.* 500500.

14. The annual rent of a certain house is $70, payable at the close of each year; what will this rent amount to in 20 years, allowing simple interest at 6 per cent.?

It is evident that the last year's rent will be $70, without interest, the last but *one* will be the amount of $70 for 1 year, the last but *two* the amount of $70 for 2 years, and so on, to the first, which will be the amount $70 for 19 years = $149.80. Therefore, *Ans.* $3198.

15. What is the amount of an annual pension of $96, remaining unpaid for 14 years, allowing simple interest, at 7 per cent.? *Ans.* $3911.04.

16. If 100 eggs were placed in a right line, exactly a yard asunder from one another, and the first a yard from a basket, what length of ground must that person go who gathers them up singly, returning with them one by one to the basket? *Ans.* 5*m.* 1300*yds*

---

17. Bought a quantity of cloth, paying 3 cents for the first yard, 5 for the second, and so on, to the last, which cost me 21 cents; how many yards did I buy?

This is the reverse of example 5; therefore $21 - 3 = 18$, $18 \div 2 = 9$, and $9 + 1 = 10$, number of yards, *Ans.*

Hence, *when the extremes and common difference are given, to find the number of terms,*

273. RULE. Divide the difference of the extremes by the common difference, and the quotient, increased by ONE, will be the answer.

18. What is the sum of the series 1, 3, 5, 7, &c. to 1001? $\overline{1001 + 1 \div 2 + 1} = 501$, number of terms. *Ans.* 251001.

19. Required the sum of the series 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, &c., to the 50th term inclusive. *Ans.* $662\frac{1}{2}$.

20. What is the sum of the descending series

$$100,\ 99\tfrac{8}{9},\ 99\tfrac{7}{9},\ 99\tfrac{6}{9},\ 99\tfrac{5}{9},\ 99\tfrac{4}{9},\ \&c,\ \textit{down to}\ 0?$$

$\overline{100 \div \frac{1}{9} + 1} = 901$, number of terms. *Ans.* 43050.

21. A person travelling from New-York to a certain place, went 3 miles the first day, 5 the second, and so on, till at last he went 58 miles in 1 day: How many days did he travel? *Ans.* 12.

THEORETICAL QUESTIONS.

What is arithmetical progression? What do the numbers form, when they *increase*? What, when they *decrease*? What are the numbers called, which form the series? What are the *first* and *last* terms called? What, the *intermediate terms*? When the first term, common difference, and number of terms, are given, how do you find the *last term*? When the extremes, and number of terms, are given, how do you find the common difference? —— the *sum of all the terms*?

GEOMETRICAL PROGRESSION.

274. GEOMETRICAL PROGRESSION is any series of numbers, continually INCREASING *by a constant multiplier*, or DECREASING *by a constant divisor*.

Thus, { 1, 2, 4, 8, 16, &c. is an increasing geometrical series.
{ 4, 2, 1, ½, ¼, &c. is a decreasing geometrical series.

275. The constant *multiplier* or *divisor* is called the RATIO.

276. In geometrical, as in arithmetical progression, there are FIVE THINGS, any *three* of which being given, the other *two* may be found.

1st. The FIRST term.
2nd. The LAST term.
3d. The NUMBER of terms.
4th. The RATIO.
5th. The SUM OF ALL THE TERMS.

1. A man bought 14 yards of cloth, and by agreement was to give what the last yard would come to, reckoning 3 cents for the first yard, 6 for the second, and so on, doubling the price to the last; what did it cost him? $3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ $245.76, *Ans.*

Here we see, that the last term is a product, of which the RATIO is thirteen times a factor, that is, *one time less* than the number of terms. The last term, then, is the 13th power of the *ratio*, multiplied by the *first term*.

But we may obtain the 13th power of the ratio, without producing all the *intermediate powers*; for $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ contains the ratio 5 times as a factor; and if we multiply 32 by *itself*, we shall evidently obtain a product of which the ratio is $5 + 5 = 10$ times a factor, that is, we shall obtain the 10th power of the ratio (= 1024); then if we multiply this 10th power by $2^3 = 2 \times 2 \times 2 = 8$, we shall obtain the $10 + 3 = 13$th power of the ratio = 8192. Hence 8192 × 3, the first term, = 24576 last term, as before. Therefore,

When the first term, ratio, and number of terms, are given, to find the last term,

277. RULE I. Write down a few *leading powers* of the RATIO with their INDICES over them.

II. Add together the most *convenient indices*, to make an index LESS BY ONE than the number of the term sought.

III. Multiply together the POWERS belonging to *those indices*, and the product multiplied by the *first term*, will be the a-

2. A boy agrees for 16 oranges, to pay only the price of the last, ing a cent for the *first*, 2 for the second, 4 for the third, and so *ling to the last; how much* did the oranges cost him. An

INDICES 1 2 3 4 5 + 5 = 10 + 5 = 15
POWERS 2, 4, 8, 16, 32, × 32 = 1024, × 32 = 32768 × 1, the first term, gives 32768 cents, for the last term = $327.68, *Ans. as before.*

3. Suppose one cent had been put out, at compound interest, in 1624, what would it have amounted to in 1828, allowing it to double once in 12 years? *Ans.* $1310.72.

4. A man plants 4 kernels of corn, which, at harvest, produce 32 kernels; these he plants the second year; now, supposing the annual increase to continue 8 fold, what would be the produce of the 19th year, allowing 1000 kernels to a pint? *Ans.* 1125899906842.624 *bushels.*

5. A butcher buys 23 oxen, and by agreement was to give what the last ox would come to, at 3 farthings for the first, 15 for the second, and so on, in 5 fold ratio; what will the oxen cost him?

Ans. £37252902984619 2*s* 9½*d.*

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6. Bought 4yds. of cloth, giving 2 cents for the first yard, 6 for the second, and so on, in 3 fold ratio; what did the whole cost me?
2 + 6 + 18 + 54 = 80 cents. *Ans.* 80 *cents.*

In a long series, the additions in this way, would become tedious; we will, therefore, try for some shorter method. Now, if all the terms of the above series, *excepting the last*, be multiplied by the ratio, the product will be the series,

6 + 18 + 54;
Subtracting the former series, - - - - - - 2 + 6 + 18,

We have, - - - - - - - - - - - - 54 − 2; that is, *the* LAST TERM *less the* FIRST TERM, which is evidently as many times the former series (2 + 6 + 18) as is expressed by the RATIO LESS ONE; whence $\frac{54-2}{3-1}$ = 26 = sum of all the terms *excepting the last*, to which, if we add the *last term*, we shall have the WHOLE AMOUNT. Thus, 26 + 54 = 80, as before.

Hence, *when the extremes, and ratio, are given, to find the sum of the series,*

278. RULE. Divide the DIFFERENCE of the *extremes* by the RATIO LESS ONE, and the quotient *increased* by the GREATER TERM will be the answer.

7. If the extremes be 4 and 274877906944, and the ratio 8, what is the whole amount of the series?

$$\frac{274877906944 - 4}{8 - 1} + 274877906944 = 314146179364, \textit{Ans}$$

8. What is the sum of the descending series 3, 1, ⅓, $\frac{1}{9}$, $\frac{1}{27}$, &c. *extended to infinity?*

It is evident the last term must become 0, or indefinitely near to nothing; consequently, the extremes are 3 and 0, and the ratio 3. *Ans.* 4½.

9. What is the value of the infinite series 4, 1, ¼, $\frac{1}{16}$, $\frac{1}{64}$, &c.? *Ans.* 5⅓.

10. What is the value of the infinite series $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c., or, which is the same thing, the repeating decimal, .1111111, &c.? *Ans.* $\frac{1}{9}$.

11. What is the value of the infinite series $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c., or of the decimal .3333333 continually repeated? *Ans.* ⅓.

12. What is the value of $\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$, &c. or of the repeating decimal .999999, &c.? *Ans.* 1.

13. What is the value of the infinite series $\frac{2}{100}$, $\frac{2}{1000}$, $\frac{2}{10000000}$, &c. or of the repeating decimal .020202, &c.? *Ans.* $\frac{2}{99}$.

14. A man bought 12 stacks of wheat, and was to give 2 cents for the first, - the second, and so on, tripling the price to the last; what

the amount of the series, we must find the last term, as above. *Ans.* $1594.

The TWO RULES, however, of finding the *last term*, and the *whole amount*, may be conveniently presented in ONE, thus:—

When the first term, the ratio, and the number of terms are given, to find the sum or amount of the series,

279. RULE. Raise the RATIO to a power whose index is equal to the number of terms, from which subtract 1; divide the remainder by the RATIO LESS ONE, and the quotient multiplied by the first term, will be the answer.

Applying the rule to the last example, $3^{12} = 531441$; then $\frac{531441-1}{3-1} \times 2 = 531440 = \$5314.40$, as before.

15. A man, dying, left his estate to his 7 sons; to the youngest, $1000, to the next, $1500, and ordered that each son should have a sum exceeding that of the next younger by the ratio $1\frac{1}{2}$; what was the whole estate?

$\frac{1.5^{7}-1}{\frac{3}{2}-1} \times 1000 = \$32171\frac{7}{8}$, or $\frac{1.5^{7}-1}{1.5-1} \times 1000 = \$32171.87\frac{1}{2}$, *Ans.*

16. A gentleman, whose daughter was married on new-year's day, gave her an English guinea, promising to double it on the first day of each month for one year; what did her portion amount to in federal money?

Ans. $19110.

17. A gentleman bought a horse, and by agreement was to give a farthing for the first nail, 3 for the second, and so on; there were 4 shoes, and in each shoe 8 nails; what did the horse cost him?

Ans. £965,114,681,693 13*s.* 4*d.*

18. A man agrees to serve a farmer 40 years without any other reward than 1 kernel of corn for the first year, 10 for the second, and so on in 10 fold ratio, till the end of the time; what will his wages amount to, allowing 1000 kernels to a pint, and supposing he sells his corn at 50 cents a bushel?

$\frac{10^{40}-1}{10-1} \times 1 = 1,111,111,111,111,111,111,111,111,111,111,111,111,111,111,111$ kernels.

Ans. $8,680,555,555,555,555,555,555,555,555,555,555,555 .555$\frac{5546875}{10000000}$.

19. The Indian who invented the game of chess, is said to have asked of his king, (who promised him any reward he should demand,) only one grain of wheat for the first point of the board, 2 for the second, and so on, in geometrical progression, for all the 64 points; how much did it amount to, allowing 700000 grains to a bushel? *Ans.* 2,635,249,153,387 *bushels*.

Compound Interest by Progression.

280. We have seen, Art. 206, that compound interest is that which arises from adding the interest to the principal at the end of each year, and making use of the amount FOR A NEW PRINCIPAL.

Hence it is obvious, that the several *amounts*, arising from any sum at compound interest, form a *geometrical series*, of which the *principal* is the FIRST TERM; number of years the *number of terms*; the *amount of* $1 or £1., &c., *at the given rate per cent.*, is the RATIO; and the *last amount* the *last term*

1. What is the amount of $5 for 7 years at 6 per cent. compound inte-

rest? or, if the first term of a geometrical series be 5, the number of terms 7, and the ratio 1.06, what is the last term?

$1.06^7 = 1.50363$, and $1.50363 \times 5 = \$7.51815$, *Ans.*

The powers of the amounts of $1, at 5 and 6 per cent. may be taken from the table, page 101.

2. What is the amount of $333.333 for 14 years, at 6 per cent. compound interest? *Ans.* $753.632+.

3. What is the compound interest of $999.999 for 19 years, at 5 per cent? *Ans.* $1526.948.

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281. The several processes may be conveniently shown by the use of letters. For this purpose, let p represent the principal, r the ratio or amount of $1, &c., for 1 year, t the time in *years*, and a the amount.

282. Two or more letters *joined together*, imply that they are to be multiplied; thus pr denotes that the *principal* is to be multiplied by the *ratio*.

283. When one letter is *placed above another, and a little to the right*, it implies that the FIRST is to be *raised to a power denoted by the* SECOND.

Thus r^t implies that the *ratio* is to be raised to a power denoted by the *time*.

4. What is the amount of $666.666 for 13 years at 5 per cent. compound interest?

RULE. $r^t \times p = a$.

Therefore, $1.05^{13} \times 666.666 = \$1257.092+$, *Ans.*

5. What is the amount of $555.555 for 15 years at 5 per cent. compound interest? *Ans.* $1154.9544.

6. What principal, will, in 7 years, amount to $7.51815, at 6 per cent. compound interest? Or, if the number of terms be 7, the ratio 1.06, and the last term 7.51815, what is the first term?

This being the reverse of the last rule, we reverse the process; that is, we change

$r^t \times p = a$ into (the) RULE. $\frac{a}{r^t} = p$.

$\frac{7.51815}{1.06^7} = \$5$, *Ans.*

7. What principal will, in 13 years, amount to $1257.09207624, at 5 per cent. compound interest? *Ans.* $666.666.

8. What is the present worth of $895.44, due 10 years hence, discounting at the rate of 6 per cent. compound interest? *Ans.* $500.

9. At what rate per cent. will $5 amount to $7.51815 in 7 years? Or, if the first term be 5, the last term $7.51815, and the number of terms 7, what is the ratio?

RULE. $\frac{a}{p} = r^t$

$\frac{7.51815}{5} = 1.50363 =$ 7th power of the ratio; whence if we extract the 7th root, we shall have the ratio $= 1.06$. *Ans.* 6 *per cent.*

Or — look in the TABLE for the power 1.50363, and at top we shall find the r· uired.

te per cent. will $400 amount to $590.98 in 8 years?

Ans. ratio 1.05 = 5 *per cent.*

11. In what time will $5 amount to $7.51815, at 6 per cent. compound interest?

RULE. $\frac{a}{p} = r^t$.

$\frac{7.51815}{5} = 1.50363 = 1.06^t$; hence if we divide 1.50363 by 1.06, and that quotient by 1.06, and so on, as long as the dvision can be made, the *number* of such divisions will be the *number of years*. *Ans.* 7 *years.*

12. In what time will $400 amount to $890.98, at 5 per cent compound interest? *Ans.* 8 *years.*

13. How long a time will it take $999.999 to amount to $2526.94747305, at 5 per cent. compound interest? *Ans.* 19 *years.*

Annuities at Compound Interest.

284. AN ANNUITY is a sum payable at regular periods of one year each, either for a certain number of years, during the life of the pensioner, or forever.

285. ANNUITIES, RENTS, &c., are said to be in ARREARS, when not paid at the time they become due.

286. The sum of all the annuities, rents, &c., remaining unpaid, together with the interest on each, for the time they have remained due, is called the AMOUNT.

1. What is the amount of an annual pension of $100, which has remained unpaid 4 years, allowing 5 per cent. compound interest?

The *last* year's pension will be $100, without interest; the last but *one* will be the *amount* of $100 for 1 year; the last but *two* the amount (compound interest) of $100 for 2 years, and so on; and the sum of these amounts will be the answer. We have then a *series of amounts* in geometrical progression, to find the sum of all the terms.

If the first term be 100, the number of terms 4, and the ratio 1.05, what is the sum of the series? See Art. 279.

$$\frac{1.05^4 - 1}{1.05 - 1} \times 100 = \$431, \textit{ Ans.}$$

Hence, *when the annuity, the time, and rate per cent. are given, to find the amount,*

287. RULE. RAISE the ratio to a power denoted by the time, from which subtract 1; then divide the remainder by the RATIO LESS ONE, and the quotient, multiplied by the annuity, will give the AMOUNT.

The powers of the amounts, at 5 and 6 per cent. may be taken from the *table*, page 101.

2. What is the amount of $200 annuity, it being in *arrears* 14 years, allowing 6 per cent. compound interest? *Ans.* $4203.

3. What is the amount of an annual rent of $150, forborne 7 years, allowing 5 per cent. compound interest? *Ans.* $1221.30.

4. At 6 per cent., what will be the amount of an annual salary of $500, for 10 years? —— for 15 years? —— for 20 years? —— for 22 years? *Ans. to last,* $21696.083⅓.

288. IF THE ANNUITY IS PAID IN ADVANCE, or if it be purchased at the beginning of the first year, the sum which ought to be paid for it is called the PRESENT WORTH.

5. What is the present worth of an annual pension of $100, to continue 4 years, allowing 5 per cent. compound interest?

The present worth is, evidently, a sum which, at 5 per cent. compound interest, would, in 4 years, produce an *amount* equal to the *amount of the annuity* in arrears the same time.

By the last rule, we find the *amount* = $431, and by directions, Art. 287, we find the present worth = $354.593+, *Ans.*

Hence, *to find the present worth of an annuity,*

289. RULE. First, find its *amount* in arrears for the whole time; then this amount divided by the *power of the ratio* denoted by the time, will give the *answer.*

6. What is the present worth of an annual salary of $500, to continue 10 years at compound interest? *Ans.* 3680.023+.

Operations in this rule being somewhat tedious, we subjoin the following

TABLE;

Showing the present worth of $1, or £1 annuity, at 5 and 6 per cent. compound interest, for any number of years under 34.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0.95238	0.94339	18	11.68958	10.8276
2	1.85941	1.83339	19	12.08532	11.15811
3	2.72325	2.67301	20	12.46221	11.46992
4	3.54595	3.4651	21	12.82115	11.76407
5	4.32948	4.21236	22	13.163	12.04158
6	5.07569	4.91732	23	13.48807	12.30338
7	5.78637	5.58238	24	13.79864	12.55035
8	6.46321	6.20979	25	14.09394	12.78335
9	7.10782	6.80169	26	14.37518	13.00316
10	7.72173	7.36008	27	14.64303	13.21053
11	8.30641	7.88687	28	14.89813	13.40616
12	8.86325	8.38384	29	15.14107	13.59072
13	9.39357	8.85268	30	15.37245	13.76483
14	9.89864	9.29498	31	15.59281	13.92908
15	10.37966	9.71225	32	15.80268	14.08398
16	10.83777	10.10589	33	16.00255	14.22917
17	10.27407	10.47726	34	16.1929	14.36613

It is obvious, that the present worth of $2 annuity will be *twice* as much as that of $1; that the present worth of $3 will be 3 times as much; and so on.

Hence, *to find, by the table, the present worth of any annuity, at 5 or 6 per cent.,*

290. RULE. Multiply the TABULAR NUMBER by the *given annuity,* and the product will be the PRESENT WORTH.

7. What is the present worth of an annuity of $30 per annum, to continue 7 years, at 6 per cent. compound interest? *Ans.* $167.4716

8. What ready money will purchase an annuity of $400, to continue 20 years at 5 per cent. compound interest? *Ans.* $4984.884.

9. At 5 per cent. compound interest, what is the present worth of an annual pension of $50, to continue 8 years? —— 10 years? —— 12 years? —— 15 years? —— 20 years? —— 25 years? —— 30 years? *Ans. to last,* $509.645

291. When annuities *do not commence* till a certain period of time has gone by, they are said to be in REVERSION.

10. What is the present worth of $100 annuity to be continued 4 years, but not to commence till 2 years hence, allowing 5 per cent. compound interest.

It is evident, the present worth will be a sum, which, in 2 years, at 5 per cent., would produce an *amount* equal to the *present worth* of the annuity, were it to *commence immediately*. By the last rule, we find the present worth of the annuity, to *commence immediately*, to be $354.595, and by directions, Art. 287, we find the present worth of $354.595 for 2 years to be $321.628+, *Ans.*

Hence, *to find the present worth of any annuity taken in* REVERSION, *at compound interest*,

292. RULE. First, find the present worth, to *commence immediately;* then THIS SUM divided by the *power of the ratio* denoted by the TIME IN REVERSION, will give the *answer*.

11. What is the present worth of a reversion of a lease of $30 per annum, to continue 7 years, but not to commence till the end of 4 years, allowing 5 per cent. to the purchaser ?

The present worth, to *commence immediately*, we find to be $173.5911; then $\frac{173.5911}{1.05^4} = \$142.814$, *Ans.*

It is plain that the same result may be obtained, by finding the present worth of the annuity to commence immediately, and to *continue to the end of the time*, that is, $4 + 7 = 11$ years, and then *subtracting* from *this sum* the present worth of the annuity, continuing for the TIME OF REVERSION.

293. Hence, we may find the present worth of $1 for the TWO TIMES *by the table*, and multiply their difference by the given annuity, and we shall have the present worth required.

Thus, $1 for the whole time, 11 years = 8.30641
For the time in reversion, 4 years = 3.54595

Difference, 4.76046
Annuity 30

$142.81380, *Ans.*

12. There is a lease of a house at $40 per annum, which is yet in being for 2 years, and the lessee is desirous to take a lease in reversion for 6 years, to begin when the old lease shall be expired, what will be the present worth of said lease in reversion, allowing 6 per cent. to the purchaser ? *Ans.* $175.0563

13. What ready money will purchase the reversion of a lease of $60, at 5 per cent., to continue 20 years, but not to commence till the end of 4 ? —— of 6 ? —— of 8 ? —— of 10 ? —— of 14 years ?

Ans. to last, $377.6556

294. PURCHASING FREEHOLD ESTATES is finding the present worth of *annuities* which CONTINUE FOREVER.

14. What is the worth of a freehold estate of $50 per annum, allowing to the purchaser 5 per cent. ?

The worth of the estate is evidently a sum, of which the yearly INTEREST is equal to the yearly RENT. The *principal* MULTIPLIED by the *rate per cent.* gives the interest; consequently the *interest* DIVIDED by the rate will give the principal.

$\$50 \div .05 = \$1000$, *Ans.*

Hence, *to find the present worth of an annuity continuing forever,*

295. RULE. Divide the annuity by the *rate per cent.*, and the quotient will be the answer.

15. If a freehold estate of $75 yearly rent, be put on sale, what is it worth allowing the purchaser 6 per cent.? *Ans.* $1250.

16. What is the present worth of $100 annuity, to continue forever, allowing the purchaser 4 per cent.? —— 5 per cent? —— 6 per cent? —— 7 per cent? —— 8 per cent? —— 10 per cent? —— 15 per cent? *Ans. to last,* $666.66⅔

17. If a freehold estate of $50 per annum, to commence 4 years hence, is to be sold, what is its present worth, allowing the purchaser 5 per cent.?

Its present worth is evidently a sum, which at 5 per cent, compound interest, would, in 4 years, produce an amount equal to the worth of the estate *if entered on immediately.*

$\frac{50}{.05} = \$1000$, the worth, if entered on immediately;

then $\frac{1000}{1.05^4} = \$822.706$, the present worth.

296. *The same* result may evidently be obtained by subtracting the *present worth* of the *annuity for the* TIME OF REVERSION, from the *worth* of the estate, to *commence immediately.*

Thus, by the *table*, the present worth of $1 for 4 years is $3.54595 \times 50 = \$177.2975$, present worth of the annuity for the *time of reversion;* then $1000 — 177.2975 = 822.7025.

18. What is the present worth of a perpetual annuity of $100 to commence 5 years hence, allowing the purchaser 5 per cent. compound interest? —— what if 8 years in reversion? —— 10 years? —— 14 years? *Ans. to last,* $1010.136

19. What is an estate of $240 per annum, worth, in ready money, to continue forever, but not to commence till the end of 3 years, at 6 per cent. compound interest? *Ans.* $3358.47¼

NOTE. The preceding examples have been confined to yearly payments; if the payments are made *half* yearly, we must take *half the principal* or *annuity, half the rate per cent.*, and *twice the number of years.*

THEORETICAL QUESTIONS.

What is geometrical progression? What is the constant multiplier or divisor called? When the first term, ratio, and number of terms, are given, how do you find the *last term?* When the extremes and ratio are given, how do you find the *sum of all the terms?* When the *first term, ratio,* and *number of terms,* are given, how do you find the AMOUNT OF THE SERIES?

What is *compound interest?* How does it appear that the *amounts,* at compound interest, form a *geometrical series?* In compound interest, what would be the *first term?* —— *number of terms?* —— the *ratio?* —— the *last term?* When the rate, time, and principal are given, how do you find the *amount?* How may the several processes be conveniently shown? When two or more letters are *joined together,* what do they imply? When *one letter is placed above another, and a little to the right,* what does it imply? When r, t, and p are given, how do you find a? When a, r, and t, are given, how do you find p? When a, p, and t, are given, how do you find r? When a, p, and r, are given, how do you find t?

What are ANNUITIES? When are they said to be in *arrears?* What is the *amount?* When the annuity, time, and rate per cent., are given, how do you find the amount? What is the *present worth* of an annuity? How do you find the present worth? —— by the TABLE? What is meant by REVERSION? How do you find the present worth of an annuity, taken in *reversion?* How, by the TABLE, do you find the PURCHASING FREEHOLD ESTATES? How do you find the present worth of a freehold estate, or an annuity continuing forever? —— the same in reversion?

The principal of any sum will double

AT	per cent. per annum, in	YEARS. at simple, and in	YEARS. at compound interest.	AT	per cent. per annum, in	YEARS. at simple, and in	YEARS. at compound interest.
3		$33\frac{1}{3}$	23.4498	6		$16\frac{2}{3}$	11.8957
$3\frac{1}{2}$		$27\frac{1}{7}$	20.1488	7		$14\frac{2}{7}$	10.2448
4		25	17.673	8		$12\frac{1}{2}$	9.0065
$4\frac{1}{2}$		$22\frac{2}{9}$	15.7476	9		$11\frac{1}{9}$	8.0432
5		20	14.2067	10		10	7.2725

MISCELLANEOUS MATTER.

CHRONOLOGICAL PROBLEMS.

297. THE GOLDEN NUMBER is a period of 19 years, in which, the changes of the moon fall on the same days of the month as before, though not at the same time of the day, there being an anticipation of 1*h.* 27′ 41″ 32‴.

To find the *Golden Number*, or *Lunar Cycle*,

298. RULE. ADD ONE to the given year; divide the sum by 19, and the remainder will be the golden number.*

EXAMPLE 1. Required the Golden Number for the year 1830.

1830 + 1 ÷ 19 = 96, and there remains 7, the *Golden Number* reqd.

2. What is the Golden Number for the year 1839? *Ans.* 16.
3. ..... for the year 1845? *Ans.* 3.
4. ..... for the year 1860? *Ans.* 18.
5. ..... for the year 1876? *Ans.* 15.

A GENERAL RULE *for finding the* EPACT FOREVER.

299. Divide the CENTURIES† of the given year by 4; multiply the remainder by 17, and to this product add the quotient multiplied by 43; divide THIS SUM plus 86 by 25, multiplying the GOLDEN NUMBER by 11, from which subtract the last QUOTIENT, and rejecting the THIRTIES, the remainder will be the EPACT.

EXAMPLE 1. Required the epact for the year 1830.

4)18
——Rem. *G. N.* = 7
Quot. 4 — 2 × 11
43 × 17 —
— — 77
172 34 *Last q.* 11—
34 —
— 66,&
206 rejecting the 30's, we have 6 for the EPACT *required.*
+ 86
—
25)292(11 *last quotient.*

2. What the epact for 1839? *Ans.* 15.
3. ..... 1845? *Ans.* 22.
4. ..... 1860? *Ans.* 7.
5. ..... 1876? *Ans.* 4.
6. ..... 1880? *Ans.* 18.
7. ..... 1888? *Ans.* 17.
8. ..... 1899? *Ans.* 18.

* If nothing remain it will be 10. † Rejecting the subsequent.

To find the MOON'S AGE on any given day,

300. RULE. To the given day of the month, add the EPACT and number of the month; and rejecting the THIRTIES, we have the MOON'S AGE.

N. B.—The numbers to be added, are—to Jan. 0, Feb. 2, March 1, April 2, May 3, June 4, July 5, Aug. 6, Sept. 8, Oct. 8, Nov. 10, Dec. 10.

EXAMPLE. For December 31st, 1829.

Given day	31	Then rejecting the 30's, we have 6 for the moon's age.
Epact	25	
No. of the month	10	For January 17th, 1830.
	—	..... November 30th, 1833.
	66	..... August 20th, 1835.
		..... February 28th, 1837.
		..... December 29th, 1840.

GRAVITIES OF BODIES.

301. The gravities of bodies above the surface of the earth *decrease* as the SQUARES OF THEIR DISTANCES IN SEMI-DIAMETERS OF THE EARTH, from the earth's centre.

1. If a body weigh 900lbs. at the surface of the earth, what will it weigh 2000 miles above the surface, estimating the earth's semi-diameter at 4000 miles? From the centre to the given height is 1½ semi-diameter; therefore $900 \div \overline{1.5 \times 1.5} = 400lbs.$, *Ans.*

2. If a body weigh 400lbs. at 2000 miles above the earth's surface, what will it weigh at the surface? *Ans.* 900*lbs.*

3. A certain body on the surface of the earth, weighs 180lbs.; how high must it be carried to weigh 20lbs.?

$\sqrt{180 \div 20} = 3$, *Ans.* 3 *semi-diameters from the earth's centre.*

4. To what height must a body be raised to lose half its weight?

$1 : \overline{3982.06}^2 :: 2 : 31713603.6872$, and $\sqrt{31713603.6872} = 5631.48$;
then $5631.48 - 3982.06 = 1649.42$ *miles*, *Ans.*

5. At what distance from the earth would a balloon be suspended between the earth and moon; the proportional quantity of matter in the earth being to that in the moon, as 41.24 to 1; and the distance of their centres $240000 + 3982.06 + 1090$?

302. RULE. As the SUM of the square roots of their QUANTITIES OF MATTER is to the DISTANCE OF THEIR CENTRES, so is the square root of the quantity of matter in the EARTH, to the distance from the EARTH'S CENTRE.

Therefore, *Ans. to the last*, 208069.43 *miles.*

303. I. If the DIAMETERS of two globes be EQUAL, and their DENSITIES DIFFERENT, the weight of a body on their surfaces will be AS THEIR DENSITIES.

II. If THEIR DENSITIES be EQUAL, and DIAMETERS DIFFERENT, the weight will be AS THEIR DIAMETERS.

III. If their DIAMETERS and DENSITIES be BOTH DIFFERENT, the weight will be AS THEIR PRODUCTS.

6. If a stone weigh 150lbs. at the surface of the earth, what will it weigh

at the surface of the sun? of Jupiter? of Saturn? of the earth, and moon? their densities being 100, 78.5, 36, 392.5, 464; and diameters 883217.58, 89170.81, 79042.35, 7964.12, and 2180 miles, *respectively.*

Ans. 4237.19*lbs.** *at the sun*, 330.61*lbs. at Jupiter*, 136.59*lbs. at Saturn, and* 48.52*lbs. at the moon.*

7. If the attraction of the moon raise a tide on the earth 5 feet, what will be the height of a tide raised by the earth on the surface of the moon, under similar circumstances?

304. The attraction of one of those bodies on the other's surface, is directly as its QUANTITY OF MATTER, and inversely as its DIAMETER.

Therefore, *Ans. to the above* 56.448 *feet.*

SPECIFIC GRAVITIES.

305. The specific gravities of bodies, are as their DENSITIES OF WEIGHTS.

Thus, if one body weighs twice as much as another of the same bulk, it is said to contain twice the specific gravity.

306. A body immersed in a fluid, will sink, if it be heavier than its bulk of the fluid, and if it be suspended therein, it will lose so much of what it weighs in air, as its bulk of the fluid weighs.

306 *a.* In finding the SPECIFIC GRAVITIES OF BODIES, the HYDROSTATIC BALANCE† is generally made use of, which differs very little from a common balance that is nicely made; only it has a hook at the bottom of each scale, on which small weights may be hung by horse hairs, so that a body suspended by the hair may be immersed in water, without wetting the SCALES.

If the body thus suspended under the scale, at one end of the balance, be first counterpoised in the air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed; then, if as much weight be put into the scale to which the body is suspended, as will restore the equilibrium, (without altering the weights in the opposite scale,) that weight which restores the equilibrium, will be equal to a quantity of water as big as the immersed body; and if the weight of the body in air be divided by what it LOSES in water, the quotient will show how much it is heavier than its bulk of water.

306 *b.* Common clear water, for common uses, is generally made a standard for comparing bodies; whose gravity may be represented by unity or 1; or, in case great accuracy is required, by 1.000, where 3 ciphers are annexed, to give room to express the ratios of other gravities in larger numbers.

NOTE. A cubic foot of common water is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or 62½ pounds.

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* Hence, it appears, that a body on the earth of 150lbs. weight, would weigh nearly two tons at the surface of the sun.

† HYDROSTATICS is the science of weighing fluids, or of weighing bodies in fluids. HYDRAULICS is the science of conveying fluids. ☞Words beginning with *hydro* or *hydra*, signify something pertaining to water, and those ending with *meter*, signify measure. HYDROMETER is, therefore, a measurer of water, or an instrument by which to measure water.

307. If the weight of a globe of any matter, in ounces, be divided by the weight of a cubic inch of that matter multiplied by .5236, the quotient will be the cube of the diameter IN INCHES.

308. The following TABLE gives the weight of a cubic foot, and also of a cubic inch, of many kinds of matter, by which, and the foregoing rule, are found the diameters of a cannon ball of 24lbs. = 5.6 inches; do. of 32lbs. = 6.18 inches; do. of 42lbs. = 6.83 inches; do. of 64 = 7.8 nearly; do. of 100 = 9.04 inches. The diameter of a ball of fine gold that will weigh a ton = 1.516 feet.*

TABLE OF SPECIFIC GRAVITIES.

A cubic foot of	Oun. Av.	c. in. oz. av.	A cubic foot of	Oz. Av.	c. in. oz. av.
Platina	20170	11.672	Brick	2000	1.1574
Fine Gold	19637	11.364	Alabaster	1875	1.085
Standard Gold	18888	10.968	Dry Ivory	1825	1.0561
Guinea Gold	17793	10.3	Brimstone	1800	1.0416
Moidore Gold	17140	9.919	Alum	1714	.992
Quicksilver	13600	7.8703	Ebony	1117	.6462
Lead	11325	6.5538	Human Blood	1054	.6199
Fine Silver	11087	6.416	Amber, Sea Water, Cow's Milk	1030	.596
Standard Silver	10535	6.096			
Rose Copper	9000	5.208	Pure Water	1000	.5787
Copper	8843	5.117	Red Wine	993	.5746
Plate Brass	8000	4.63	Oil of Amber	978	.566
Steel	7852	4.544	Proof Spirits,	925	.5353
Cast Brass	7850	4.542	Dry Oak	do.	do.
Iron wrought	7645	4.424	Olive Oil	913	.5289
Block Tin	7321	4.236	Loose Gunpowder	872	.504
Cast Iron	7135	4.13	Spirits of Turpentine	864	.5
Diamond	3490	1.968	Alcohol or Pure Spirit	850	.4919
Crystal Glass	3150	1.823	Elm and Ash	800	.463
White Marble	2707	1.566	Oil of Turpentine	772	.4468
Rock Crystal	2658	1.538	Dry Crabtree	765	.4427
Green Glass	2620	1.516	Ether	732	.4236
Clear Glass	2600	1.5	White Pine	569	.3292
Stone (Flint)	2582	1.494	Sassafras	482	.279
Stone (Paving)	2570	1.487	Cork	240	.139
Stone (Cornelian)	2568	1.486	Common Air	1.25	.0007
Stone (Free)	2352	1.361	Inflammable Air	0.13	.00007

NOTE 1. The specific gravity of any body, that of water being 1, may be readily found by pointing off 3 places from the right hand of the given weight. Thus, the specific gravity of standard gold is 18.888.

2. All bodies expand with heat, and contract with cold; consequently, their specific gravities are not precisely the same in summer as in winter.

309. How to discover the QUANTITY OF ADULTERATION IN METALS.

8. Suppose the specific gravity of a body, compounded of gold and silver, be 14; that of standard silver, (by the table,) is 10.535, and that of standard gold 18.888, it is required to find the quantity of each metal in the compound.

* Let the learner obtain these results.

14 — 10.535 = 3.465, the proportional BULK of gold in the compound; and 18.888 — 14 = 4.888, the proportional bulk of silver. Then, 18.888 × 3.465 = 65.447, for the proportional WEIGHT of gold; and 10.535 × 4.888 = 51.495, for the proportional weight of silver.

Lastly, as $65.447 + 51.495 : \left\{\begin{matrix}65.447\\51.495\end{matrix}\right\} :: 14 :: \left\{\begin{matrix}7.8351 \text{ of gold.}\\6.1649 \text{ of silver.}\end{matrix}\right.$

Hence it will be easy to tell whether any suspected metal be genuine or alloyed.*

310. HOW TO TRY SPIRITUOUS LIQUORS. A cubic inch of good brandy, rum, or other proof spirits, weighs 234 grains; therefore, if a true inch cube of any metal weigh 234 grains less in spirits than in air, *it shows that the spirits are* PROOF.

If the cube lose less of its aerial weight in spirits, they are above proof; if it lose more they are under proof; for the better the spirits are, the lighter they are, and the worse the heavier.

9. The cubic inch of common glass weighs about 1.86oz. Troy; do. of salt water .5427oz., do of brandy .48927oz. Suppose then, a seaman has a gallon of brandy in a bottle, which weighs 4½lbs. Troy, and to conceal it, throws it overboard into salt water; will it sink or swim, and by how much is it heavier or lighter than its bulk of salt water?

Ans. it will sink, being 20.11oz. heavier than its bulk of salt water.

Given the WEIGHT *to be raised by a balloon to find its* DIAMETER.

311. RULE. As the specific difference between common and inflammable air, is to one cubic foot; so is any weight to be raised, to the cubic feet contained in the balloon.

Whence the diameter may be found by the note at the bottom of page 128.

10. Required the diameter of a spherical balloon of sufficient capacity to ascend with 4 persons weighing on an average 160lbs each, and the balloon and a bag of sand weighing 60lbs. *Ans.* 26.65 *feet.*

Given the diameter of a balloon to find WHAT WEIGHT *it will raise.*

312. RULE. As one cubic foot is to the specific difference between common and inflammable air; so is the CAPACITY of the balloon to the weight it will raise.

11. What weight will a balloon raise, whose diameter is 26.65 feet?

$26.65^3 \times .5236 = 9911.4+$, its capacity; therefore, *Ans.* 700*lbs.*

313. The magnitude of any body multiplied by its specific gravity, gives its absolute weight.

12. What weight of lead will cover a house, the area of the roof being 6000 feet, and the thickness of the lead $\frac{1}{12}$ of a foot? *Ans.* 35390*lbs.* 10*oz.*

314. The *weight* of any body divided by its *specific gravity* gives ITS MAGNITUDE.

13. What is the magnitude of 1oz. of clear glass? *Ans.* 8.64 *cubic in.*

315. The weight divided by the magnitude of any body gives its *specific gravity.*

14. A piece of marble, containing 8 cubic feet, weighs 1853¼lbs.; what is its specific gravity? *Ans.* 3707*oz.*

To find the quantity of pressure against a SLUICE *or* BANK *which pens water.*

316. RULE. Multiply the area of the sluice, by the depth

* The question concerning king Hiero's crown, may be wrought by this process; and *perhaps this was the way* in which Mr. Thompson worked it in the rule of three.

of the centre of gravity IN FEET,* and this product again by 62½, and it will give the POUNDS *required.*

15. What is the pressure against a sluice or flume, 30 feet long, 5 feet wide, and 4 feet deep? *Ans.* 18750*lbs.*

THE FALL OF BODIES.

317. The velocities† of FALLING BODIES, *are as the* SQUARES OF THE TIMES *in which they fall;* or, which is the same thing,

THE DISTANCE FALLEN THROUGH IN FEET, IS ALWAYS EQUAL TO THE SQUARE OF THE TIME IN FOURTHS OF A SECOND.

1. A bullet being dropped from the top of a building, was found to reach the ground in 1¾ seconds; required its height.

1¾ = 7 *fourths*, and 7 × 7 = 49 feet, *Ans.*

2. How far will a bullet fall in 11¾ seconds? *Ans.* 2209 *feet.*

3. What is the difference between the depths of two wells, into each of which should a stone be dropped in the same instant, one would reach the bottom in 2 seconds, and the other in 3? *Ans.* 80 *feet.*

4. Suppose a ball to be raised so high that it will be 1 minute in falling, how far will it fall in the last second of time? *Ans.* 1904 *feet.*

318. The SQUARE ROOT OF THE FEET, in the SPACE FALLEN THROUGH, will ever be equal to ONE EIGHTH of the velocity acquired at the END OF THE FALL. Hence,

To find the space fallen through, the velocity being given,

RULE. Divide the VELOCITY by 8, and the SQUARE OF THE QUOTIENT will be the distance fallen through, to acquire that velocity.

5. The velocity of a cannon ball being about 660 feet per second; how high must a body fall, to acquire the same velocity?

660 ÷ 8 = 82.5, and 82.5 × 82.5 = 6806.25 feet, *Ans.*

319. ASCENDING BODIES ARE RETARDED IN THE SAME RATIO THAT DESCENDING BODIES ARE ACCELERATED.

6. If a ball discharged from a gun, return to the earth in 12 seconds, how high did it ascend?

The ball is half the time ascending, 6 × 4 = 24, then 24 × 24 = 576 feet, *Ans.*

320. FOUR TIMES the number of SECONDS, in which a body has been falling, is equal to ONE EIGHTH of the velocity, in feet, per second, acquired at the end of the fall. Hence,

To find the time, the velocity per second being given,

RULE. Divide the GIVEN VELOCITY by 8, and ONE FOURTH PART of the quotient will be the answer.‡

7. How long must a bullet be falling to acquire a velocity of 160 feet per second? 160 ÷ 8 = 20, and 20 ÷ 4 = 5 seconds, *Ans.*

8. ——— 440 feet per second? *Ans.* 11 *seconds.*

321. It is plain, we may find the time a body has been falling, having the space fallen through given, by reversing principles Art. 317, that is, *by taking* ONE FOURTH PART OF THE SQUARE ROOT *of the given number of feet.*

9. In how long time will a bullet fall through a space of 15625 feet?

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* of gravity is equal to half the depth of the fluid.

† increases as the odd numbers, 1, 3, &c. each quarter of a second.

$\sqrt{15625} = 125$, and $125 \div 4 = 31.25$ seconds, *Ans.*

10. In what time will a musket ball, dropped from the top of a steeple, 484 feet high, come to the ground? *Ans.* 5½ *seconds.*

322. *To find the velocity, per second, with which a heavy body will begin to descend, at any given distance above the earth's surface,*

RULE. As the square of any given distance from the earth's centre: is to the square of the earth's semidiameter; : so is 16 feet: to the velocity required.

11. With what velocity, per second, will an iron ball begin to descend if raised 3000 miles above the earth's surface?

$\overline{4000 + 3000} \times \overline{4000 + 3000} : 4000 \times 4000 :: 16 : 5.22449$ feet, *Ans.*

323. It is evident we may find the MEAN VELOCITY of a falling body, *by dividing the* SPACE FALLEN THROUGH *by the number of seconds* it has been falling.

12. A musket ball dropped from the top of a steeple 484 feet high in 5½ seconds; required its mean velocity. *Ans.* 88 *feet per second.*

324. *To find the velocity acquired by a falling body per second, (or by a stream of water, having the perpendicular distance given,) at the end of any given time,*

RULE. The VELOCITY acquired at the END OF ANY PERIOD is equal to twice the *mean velocity* during that period.

Or, if we multiply the perpendicular space fallen through by 64, the square root of the product will be the velocity required.

13. If a ball fall through a space of 484 feet in 5½ seconds, with what velocity will it strike? $484 \div 5.5 = 81$, and $88 \times 2 = 176$, *Ans.*

Or $484 \times 64 = 30976$, and $\sqrt{30976} = 176$ as before.

14. One end of a sluice or flume is 2½ feet lower than the other; required the velocity of the stream per second? *Ans.* 12.649*ft.*

325. *The velocity, with which a falling body strikes, given to find the space fallen through.* This is evidently the reverse of the above, wherefore we have this

RULE. Divide the square of the velocity by 64, and the quotient will be the height required.

15. If a ball strike the ground with a velocity of 56 feet per second, from what height did it fall? $\overline{56 \times 56} \div 64 = 49$ feet, *Ans.*

16. The mean velocity of a fluid or stream, is 12.649 feet per second; required the perpendicular fall of the stream. *Ans.* 2½ *feet.*

326. THE MOMENTUM OR FORCE, with which a falling body strikes, *is equal to its* WEIGHT *multiplied by its* VELOCITY.

17. If the rammer, used for driving the piles of Charlestown bridge, weighed 2½ tons, or 4500 lbs., and fell through a space of 10 feet, with what force did it strike the pile?

By Art. 324, $\sqrt{10 \times 64} = 25.3 =$ the velocity, and $25.3 \times 4500 =$ 113850 lbs. momentum, *Ans.*

327. Hence we may find the *space fallen through*, by dividing the momentum by the weight, for the velocity; and then the square of the velocity by 64.

18. If the aforementioned rammer weighed 4500lbs., and struck with a force of 113850lbs.; from what height did it fall? *Ans.* 10 *feet.*

328. When it is required to know with what quantity of motion, momentum or force, a fluid, moving with a given velocity, will strike against a fixed obstacle, the result may be obtained by the following

RULE. Multiply the height of the fall by 62.5 lbs. Avoir. for clear water, by 63 for dirty water, and by 64 for sea water.

19. Suppose a stream of clear water to move at the rate of 5 feet per second, and to meet with a fixed obstacle (or bulk head) 15 feet wide, and 4 feet high; required the momentary pressure of the stream. $\overline{5 \times 5} \div 64 = .39$ of a foot, the perpendicular fall of the water (Art. 325.) Then $52.5 \times .39 = 24.375$ lbs. pressure upon each square foot, which multiplied by 60 ($= 15 \times 4$) gives 1462.5 lbs.; and this by the given velocity, produces 7312.5 lbs., *Ans.**

329. The velocity of water, spouting through a sluice, or aperture in a reservoir, or bulkhead, is the same that a body would acquire by falling through a perpendicular space equal to that between the *top of the water* and the *aperture.*

20. What is the velocity of water, issuing from a head of 5 feet deep?

$64 \times 5 = 320$, and $\sqrt{320} =$ nearly 18 feet, *Ans.*

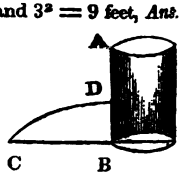
21. If the velocity of a stream issuing through the bulkhead of a mill, be 16 feet per second, what head of water is there? *Ans.* 4 *feet.*

330. The quantity of water discharged from a hole in a vessel, is *as the* SQUARE ROOT *of the height of water above the* APERTURE.

22. A miller has a head of water 4 feet above the sluice; how high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time?

$\sqrt{4} = 2$, then $2 + \frac{1}{2}$ of $2 = 3$, and $3^2 = 9$ feet, *Ans.*

331. If a large vessel be kept constantly full, it is required to find how far water will spout from it through a hole D, on the plane B C?



RULE. B C will ever be equal to twice the SQUARE ROOT of A D × D B.

NOTE. The fluid will spout farthest, if the hole be made half way between A and B.

23. How far will water spout on the plane B C, if the distances between the top and the hole, and bottom and the hole be 2 and 6? —— 3 and 5? —— 4 and 4? —— 5 and 3? *Ans. to last,* 7.7459666.

332. If the vessel be raised above the plane, the foregoing rule and note will apply equally well, only we must regard the distances given, *as between the* TOP *and the orifice, or hole, and between the* PLANE *and the hole.*

24. A vessel 10 feet deep, is raised 2 feet above a surface or plane; it is required to find how far water will spout through a hole 3 feet from the bottom? *Ans.* 11.8321596 *feet.*

333. I. If the WEIGHTS of any two bodies, put in motion, be equal, the FORCES by which they are moved, will BE AS THEIR VELOCITIES.

II. If the VELOCITIES be equal, their FORCES will be AS THEIR WEIGHTS.

* Water, being a yielding substance, is said to lose $\frac{1}{3}$ of its power in producing effects.

III. If both their WEIGHTS and VELOCITIES be unequal, THE FORCES will be AS THEIR PRODUCTS.

25. If a body weighing 30lbs. be impelled by such a force as to send it 20 rods in a second, with what velocity would a body weighing 12lbs. move, if impelled by the same force? *Ans.* 50 *rods per sec.*

26. Suppose the battering ram of Vespasian weighed 60000lbs.; that it was moved at the rate of 24 feet per second; and that this was sufficient to demolish the walls of Jerusalem: with what velocity must a cannon ball of 42lbs., be moved to do the same execution?

Ans. 34285$\frac{5}{7}$ *ft. per sec.*

MECHANICAL POWERS.

334. These are usually accounted SIX in number, namely, the LEVER, the WHEEL and AXLE, the PULLY, the INCLINED PLANE, the WEDGE, and the SCREW.

335. WEIGHT and POWER, when opposed to each other, *signify the body* TO BE MOVED, *and the body that* MOVES IT.

336. It is a PRINCIPLE in mechanics, *that the* POWER *is to the* WEIGHT, *as the* VELOCITY OF THE WEIGHT *to the* VELOCITY OF THE POWER.

THE LEVER OR STEELYARD.

337. The distance between the *body to be raised or balanced*, and the *fulcrum* or *prop*, is to the distance between the *prop* and *the point where the power is applied*, as the POWER is to the WEIGHT it will balance.

1. If a man of 160lbs. weight, rest on the end of a lever 10 feet long, what weight will he balance on the other end, supposing the prop one foot from the weight? *Ans.* 1440*lbs.*

2. In giving directions for making a chaise, the length of the shafts between the axletree and backbond being settled at 9 feet, a dispute arose whereabout on the shaft the centre of the body should be fixed. The chaise maker advised to place it 30 inches before the axletree; others supposed 20 inches would be sufficient: now, supposing two passengers to weigh 3cwt., and the body of the chase $\frac{3}{4}$cwt. more; what will the beast in each of these cases bear more than his harness?

Ans. 116$\frac{2}{3}$*lbs. in the former, and* 77$\frac{7}{9}$ *in the latter.*

THE WHEEL AND AXLE.*

338. The proportion for the wheel and axle, (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel turns round,) is, *as the diameter of the* AXLE *is to the diameter of the* WHEEL, *so is the* POWER APPLIED TO THE WHEEL, *to the* WEIGHT SUSPENDED *by the axle.*

* In this kind of machinery it is usual to abate $\frac{1}{3}$ *of the effect for friction.*

3. A mechanic would make a windlass in such a manner, that 11lb. applied to the wheel, shall be equal to 101bs. suspended from the axle; now, supposing the axle to be 6 inches, required the diameter of the wheel?

Ans. 60 *inches.*

THE PULLY.

339. If a *power* sustain a *weight* by means of a FIXED PULLY, the *power and weight are* EQUAL; but if the pully be *moveable*, the power is but HALF the weight; hence for EVERY *cord passing over a moveable pully, the power is* DOUBLE.

4. What weight will a person of 160lbs. raise by means of a rope passing over three fixed and three moveable pullies alternately?

Ans. 960*lbs.*

THE INCLINED PLANE.

340. The power gained by the inclined plane is, *as the* LENGTH *of the plane to its* PERPENDICULAR HEIGHT.

NOTE. The double of the inclined plane forms the WEDGE. To obtain the greatest effect by this power, the body must be moved in a line parallel to the inclination.

5. What weight will a power of 150lbs. draw up an inclined plane, whose height is 10 feet, and length 30 feet?

Ans. 450*lbs.*

THE WEDGE.

341. When a wedge is in equilibrio, *the power acting against the* BACK, *is to the force acting* PERPENDICULARLY *against* EITHER SIDE, *as the* BREADTH OF THE BACK *to the* LENGTH OF THE SIDE.*

NOTE. The above respects only *one* side; to calculate for both sides, we must double the result.

6. How large a mallet will it take to raise a ship of ten tons, by means of a *half wedge*, whose back is 5 inches, and inclined side 20, supposing it to be impelled at the rate of 200 feet per second?

Ans. allowing for friction, it will take a mallet of 56*lbs. weight.*

THE SCREW.

342. The POWER is to the WEIGHT to be raised as the distance between TWO THREADS of the screw, to the CIRCUMFERENCE OF A CIRCLE described by the POWER applied at the END OF THE LEVER.

7. What power applied at the end of a lever 5 feet long, will raise 9 tons, by means of a screw whose threads are an inch asunder?

Abating ½ *for friction, Ans.* 71.3 + *lbs.*

PENDULUMS.

343. The time of a vibration, is to the time of a heavy body's descent through half the length of the pendulum, as the circumference of a cir-

* *Here the friction is very great; and is, at least, equal to the force to be overcome.*

cle to its diameter. Therefore, (as a body descends freely through the space of about 193.5 inches in one second of time,)

To find the length of a pendulum vibrating seconds,

RULE. As $\overline{3.1416}^2$: to 1×1 : : so is 193.5 : to 19.6, the HALF LENGTH, which multiplied by 2, gives the LENGTH = 39.2 inches.

344. *To find the length of a pendulum that will swing any given time,*

RULE. Multiply the square of the time in seconds by 39.2, and it will give the length in *inches*.

1. What are the lengths of pendulums, vibrating ¼ seconds, ½ seconds, seconds, minutes, and hours? *Ans. to last,* 508032000 *inches.*

2. What is the difference between two pendulums, the one vibrating half seconds, and the other 3 seconds? *Ans.* 343 *feet.*

345. *To find the time a pendulum of any given length will swing,*

RULE. The SQUARE ROOT of the given length divided by 39.2, will give the TIME IN SECONDS.

3. How often will a pendulum of 2.45 inches vibrate in a second? *Ans.* 4 *times, that is, it will vibrate* ¼ *seconds.*

346. *To find the true depth of a well by dropping a stone into it.*

RULE I. With A PENDULUM find the time from the dropping of the stone, till you hear it strike the bottom.

II. Multiply 73088 (= $16 \times 4 \times 1142$) by the NUMBER OF SECONDS thus found.

III. From the SQUARE ROOT of this product plus 1142^2 take 1142.

IV. The SQUARE of the remainder divided by 64, will give the depth of the well IN FEET.

NOTE. The depth of the well divided by 1142, will give the time of the sound's ascent, which taken from the whole time will leave the time of the stone's descent.

4. When I dropped a stone into a well, a line with a bullet which measured to the middle of the ball 2.45 inches, made 8 vibrations before I heard the stone strike the bottom; what was the depth of the well? —— time of the stone's descent? and of the sound's ascent?

Ans., the depth of the well was 60.645 *feet; stone's descent* 1.947, *and sound's ascent* .053 *of a second.*

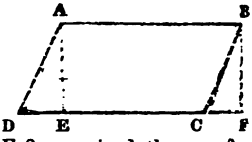
MENSURATION
OF
SUPERFICIES AND SOLIDS.

(See pages 35, 49, 50, 116, 119, & 124.)

347. *To measure a* RHOMBOID.

DEF. A RHOMBOID is a figure, *whose opposite sides and opposite angles are* EQUAL.

For measuring a figure in this form, it is plain we have this



RULE. Multiply ONE OF THE LONGEST SIDES by the PERPENDICULAR DISTANCE.

For, by the figure, A E D will fill B F C.

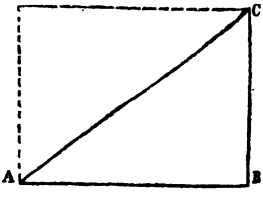
Ex. 1. Let A B or D C be 10 rods, and A E 8; required the area? $10 \times 8 = 80$ square rods, *Ans.*

2. Let the longer sides be 40 rods each, and perpendicular distance 39 rods; required the number of square rods? *Ans.* 1560.

3. How many acres in the above? *Ans.* 9*A.* 3*R.*

4. If the longer sides of a rhomboid be 50 rods each, and perpendicular 47, what is the area in acres, &c.? *Ans.* 14*A.* 2*R.* & 30*sq. rods.*

348. *To measure a* RIGHT-ANGLED TRIANGLE.



RULE. Multiply the BASE* by the PERPENDICULAR, and it will give the DOUBLE AREA.

5. Given the base AB = 8 rods, and perpendicular BC 6, to find the area. $8 \times 6 = 48$, and $48 \div 2$ gives 24 sq. rods, *Ans.*

6. Let the base be 18 rods, and perpendicular 12; required the area. *Ans.* 2*R.* 29*rds.*

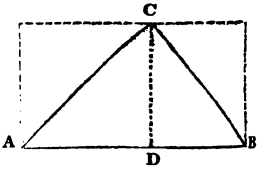
7. Base 78, and perpendicular 45. *Ans.* 10*A.* 3*R.* 35*rds.*

349. *To measure an* OBLIQUE-ANGLED TRIANGLE.

DEF. 1. A TRIANGLE is OBLIQUE, when not RIGHT-*angled.*

2. ANGLES are said to be OBTUSE when *greater than right angles*, and ACUTE when *less.*

Wherefore it is plain we have this



RULE. Multiply the BASE by the PERPENDICULAR, and it will give TWICE THE AREA of the triangle.†

8. Given the side AC = 26, the side BC 22, and the side AB 40, to find the area of the triangle.

Before proceeding to apply the rule, we must evidently find the perpendicular CD; and in doing this, we must first find where, on the line AB, the perpendicular will fall. This may be done after the FOLLOWING MANNER:

349 *a.* As the WHOLE BASE : is to the SUM OF THE TWO SHORTER SIDES : : so is the DIFFERENCE of those sides : to the DIFFERENCE OF THE

---

* The longest side of a triangle is usually called the base, except in a right angled triangle, where the longest of the two legs is called the base.

† The following is *another* method of finding the area of a triangle:

From the HALF SUM *of the three sides, take* EACH SIDE *severally, and note* THE DIFFERENCES; *then multiply the* SAID HALF SUM *and* DIFFERENCES *continually together, and the* SQUARE ROOT *of their product will be the* AREA REQUIRED.

It will be very good exercise for the learner to prove the above examples by this rule.

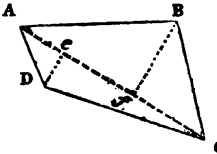
SEGMENTS* of the base; then, by adding half this difference to half the base, we shall have the longer segment AD; and by subtracting said half difference from half the base, we have the shorter segment DB. After this is done, we may find the perpendicular by rule 2, Art 259.

Applying the rule, $26 + 22 = 48$, and $26 - 22 = 4$; then, as $40 : 48 :: 4 : 4.8$, and $\frac{1}{2}$ of $40 + 2.4$ $(= 4.8 \div 2) = 22.4$, the longer segment. Again, $\frac{1}{2}$ of $40 - 2.4 = 17.6$, the shorter segment. And by the rule, Art. 259, $\sqrt{22^2 - 17.6^2} = 13.2$, the perpendicular. By the same rule, $\sqrt{26^2 - 22.4^2} = 13.2$, as before. Consequently, $\overline{40 \times 13.2} \div 2 = 132$, the area of the triangle.

9. What is the area of a triangle, the three sides being 13, 11, and 18? —— 20, 30, & 50? —— 26, 22, and 40? —— 32, 51, and 60? —— 33, 45, and 52? ——63, 19, and 70? —— 72, 35, and 80?
Ans. to the last, 1259.997984 *rods.*

350. *To measure a* TRAPEZIUM.

DEF. A TRAPEZIUM is a figure *of four* UNEQUAL SIDES, *and* UNEQUAL *angles.*

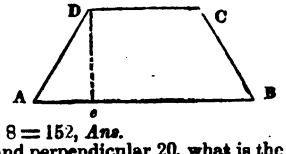


RULE. Dividing the trapezium as represented in the figure, we have *two triangles,* of which, *the* SUM *of the perpendiculars, multiplied into the* DIAGONAL, *gives the* DOUBLE AREA.

10. Given the perpendiculars De & $Bf = 6$ and 10, and diagonal AC 24, to find the areas. $6 + 10 = 16$, then $16 \times 24 = 384$, and $384 \div 2 = 192$, *Ans.*
11. If the perpendiculars be 19 and 27, and diagonal 45, what is the area? *Ans.* 877.5.
12. Given the perpendiculars = 35 and 17, and diagonal 63, to find the area? *Ans.* 1638.
13. How many acres in the last, allowing the dimensions in rods? *Ans.* 10*A.* & 38*rds.*

351. *To measure a* TRAPEZOID.

DEF. A TRAPEZOID is the *segment of a triangle,* cut by a line parallel to the base.



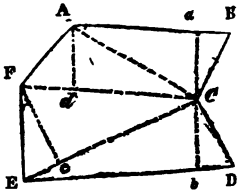
RULE. Multiply HALF THE SUM of the parallel sides by the PERPENDICULAR DISTANCE.

14. Given the side AB = 25, the side CD 13, and perpendicular 8, to find the area. $\overline{25 + 13} \div 2 = 19$, and $19 \times 8 = 152$, *Ans.*
15. If the parallel sides be 35 and 17, and perpendicular 20, what is the area? *Ans.* 250.
16. The two parallel sides being 79 and 50, and perpendicular 35, what is the area? *Ans.* 2257½.

* A *segment* or *frustum* signifies *a* PIECE *or* PART *of a thing.*

352. *To find the area of an* IRREGULAR FIGURE,

RULE. Divide the FIGURE into triangles, then measure EACH OF THE TRIANGLES, and the SUM of their SEVERAL AREAS, will be the AREA REQUIRED.



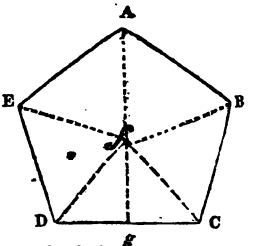
The figure ABCDEF being given, divide it into triangles by the diagonals AC, FC, and EC: then these triangles may be measured by letting fall perpendiculars, as A*d*, F*e*, C*a*, C*b*, and *multipling these perpendiculars, by* HALF THEIR RESPECTIVE BASES.

17. In the triangle ABC the base AB is 50, and perpendicular 19; in the triangle FCA the base FC is 52, and perpendicular 21; in the triangle ECF the the base EC is 55, and perpendicular 23; and in the triangle EDC the base ED is 59 and perpendicular C*b* 20; required the area of the figure ABCDEF. *Ans.* 2243.5.

353. *To measure a* REGULAR POLYGON.

DEF. A REGULAR POLYGON is a figure *whose sides and angles are* ALL EQUAL.

RULE. Multiply the length of ONE SIDE, by HALF THE NUMBER OF SIDES, and that product by the PERPENDICULAR.



18. Required the area of the five sided figure ABCDE, each side of which being 25 and perpendicular *fg* 17.2047737.

$25 \times 2\frac{1}{2} = 62\frac{1}{2}$; then $62\frac{1}{2} \times 17.204773 = 107.298356$, *Ans.*

The names of several polygons, together with their superficial contents, (*the sides being* ONE,) may be seen in the following

TABLE.

Number of sides	Names	AREAS OF Multipliers.	Number of sides.	Names.	AREAS OF Multipliers.
3	Trigon.....	.433013	8	Octagon.....	4.828427
4	Tetragon..	1.	9	Nonagon....	6.181824
5	Pentagon..	1.720477	10	Decagon....	7.694209
6	Hexagon...	2.598076	11	Endecagon	9.365639
7	Heptagon..	3.633912	12	Dodecagon.	11.196152

Hence, if we square the side of a polygon, and multiply it by the MULTIPLIER, it will give the AREA.

354. *To find the area of a* CIRCLE.*

RULE. MULTIPLY HALF THE DIAMETER INTO HALF THE CIRCUMFERENCE.

* 1. Diam. × 3.14159, or ÷ .31831, gives the circumference.
2. Circum. × .31831, or ÷ 3.141659, gives the diameter.
3. Diam. × .886227, or ÷ 1.128379, gives the side of an equal square.
4. Diam. × .866024, or ÷ .1547, = the side of an inscribed equilateral triangle.
5. Diam. × .70710678, or ÷ 1.414203, gives the side of an inscribed square.
6. Diam.^{2} × .785398, or ÷ 1.273241, gives the area of the circle.
7. Circum. × .282094, or ÷ 3.544907, gives the side of an equal square.

19. Required the area of a circle, whose diameter is 7 feet, and circumference 22? *Ans.* 38½*sq. ft.*

355. As 7 is to 22, or *more exactly*, as 113 to 355, or *by decimals*, as 1 to 3.1416, so is the DIAMETER OF A CIRCLE to its CIRCUMFERENCE.

20. What is the circumference of a circle, whose diameter is 12? *Ans.* 37.71, 37.699, *or* 37.6992.

21. The diameter of a circle, whose circumference is 12? *Ans.* 3.8+.

22. What is the area of a circle, drawn within a square, containing 1 square rod?

The diameter of the circle will be 1 and circumference 3.1416; therefore, *Ans.* 7854 *nearly.*

356. Hence, *if the diameter of a circle be given, to find the* AREA,

RULE. MULTIPLY THE SQUARE OF THE DIAMETER BY .7854.

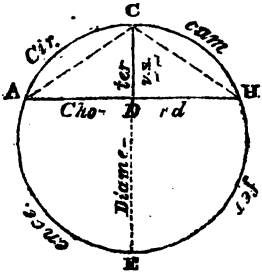
23. What is the area of a circle, whose diameter is 12? *Ans.* 113.0976.

24. ——— whose diameter is 113 rods? *Ans.* 62*A.* 2*R.* 28.77*sq. rods.*

NOTE 1. If the area be given to find the diameter, reverse the process.

2. The area of a half or semicircle is one half the area of the whole circle; and that of a quadrant is one fourth.

357. 25. Given the chord AH = 16 chains, and the chord AC = 10, to find the length of the arch line, ACH.



RULE. From TWICE THE CHORD AC, take the CHORD AH, and ⅓ of the remainder added to TWICE AC, will give the ANSWER.

10 × 2 = 20, and 20—16 = 4; then ⅓ of 4 + 20 = 21⅓ chains, *Ans.*

26. If, in the segment AHC, the chord AH be 216, do. of AC 126, what is the length of the arch line? *Ans.* 264.

358. 27. Given the versed sine DC = 6 chains, and the chord AH 16, to find the diameter CE.

RULE. Divide the SQUARE of HALF THE CHORD by the VERSED SINE, and add the QUOTIENT to the SINE.

$8^2 \div 6 = 10.666$ &c. which added to 6, gives 16.66666 &c. chains, *Ans.*

28. If the chord AB be 1869.5, and versed sine 423.5, what is the diameter? *Ans.* 2486.6+.

359. Given the SECTOR of a circle, as AEBC (next page), to find the area.

RULE. Multiply the RADIUS OR HALF DIAMETER, by HALF THE ARCH LINE.

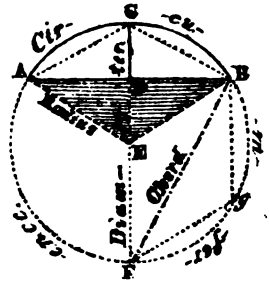
29. Given radius AE = 72, chord AB = 126, and AC = 70, to find the area of the sector AEBC. *Ans.* 5207.76.

8. Circum. × .2756646, or ÷ 3.6275939, gives the side of an equilateral triangle inscribed.
9. Circum. × .225079, or ÷ 4.442877, gives the side of an inscribed square.
10. Circum.^{2} × .079577525, or ÷ 12.56636217, gives the area of the circle
11. Area × .1273241, or ÷ .785398, gives the square of the diameter.
12. Area × 12.56636217, or ÷ .079577525, gives the square of the circumference.
13. When the diameter of one circle is 1, and the diameter of another 2, the circumference of the first is equal to the area of the second, = 3.141592.
14. If the circumference be 4, the diameter and area are equal, = 1.273241.
15. If the diameter be 4, the circumference and area are equal, = 12.566368.

30. If radius AE be 8½ chains, chord AB 16, and AC 10, required the area of the sector AEBC. *Ans.* 88.888888, &c. *square chains*

31. If the chord Bg be 126, do. the chord BF 204, and radius EB or EA be 112, what is the area of the sector AEBF? *Ans.* 28224

360. To find the area of a SEGMENT OF A CIRCLE, as ADBCA, or ADBFA.



RULE. Find the area of the SECTOR, then add or subtract the area of the TRIANGLE formed by the CHORD and TWO RADII, as the case may require.

32. Required the area of the segment ADBC, whose chord AB is 172, do. AC or CB 104, and versed sine 58.49.

NOTE. Here the area of the triangle must be subtracted from that of the sector.

Ans. 7248.25.

33. If the chord AB, of the segment ADBFA, be 136 links, do. of BF 146, of Bg or gF 86, and radius 80, what is the area in acres, &c.

NOTE. Here the area of the triangle must be added.

Ans. 17*A.* 1*R.* 9.48*rds*

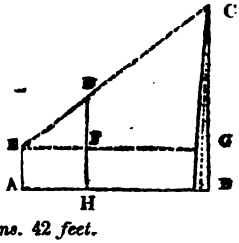
361. *To measure an* ELLIPSIS *or* OVAL.

DEF. AN ELLIPSIS OR OVAL is a LONG ROUND, having two diameters; the longest of which is called the TRANSVERSE, and the shortest the CONJUGATE.

RULE. The rectangle of the TWO DIAMETERS multiplied by .7854, will give the AREA required.

34. Required the area of an ellipsis, whose transverse and conjugate diameters are 88 and 72. *Ans.* 4976.29+.

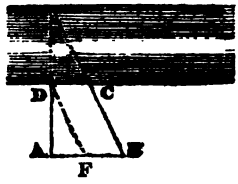
362. *To measure the height of a* TREE *or* OTHER INACCESSIBLE OBJECT.



RULE. Set TWO STAKES at A and H, PARALLEL TO THE BODY OF THE TREE, and ranging with the top C; then say, as AH : FD : : AB : GC, which added to BG, gives the height required.

35. If the shorter stake be 6 feet, distance between them 10, diff. of their lengths 8, and distance between the tallest and the tree be 35; required the height of the tree. *Ans.* 42 *feet.*

363. *To find the breadth of a* RIVER, *or* INACCESSIBLE MOAT.



RULE. Begin at the BANK OF THE RIVER D, opposite some OBJECT (E), so that DA may be at RIGHT ANGLES with the bank of the river DC; and measure off any convenient distance to A; thence to B, making a right angle at A; thence (towards E) to the bank of the river at

C; thence to D again, putting a stake at D, A, B, and C respectively. Then, AB — DC = AF; and AF: AD:: DC: DE.

36. If DA be 10 chains, AB 13, and DC 9; what is the breadth of the river DE? *Ans.* 22.5

MENSURATION OF SOLIDS.

(See pages 59, 116, and 124.)

364. *To measure a* PARALLELOPIPED.

DEF. A PARALLELOPIPED is a solid of three dimensions; and differs from a CUBE, in that, *its three dimensions are* NOT ALL EQUAL.

RULE. Multiply the THREE DIMENSIONS TOGETHER *continually*.

37. How many cubic inches in a box 3ft. 5in. wide, 3ft. 9in. high, and 4ft. 10in. long? *Ans.* 107010.

38. How many bushels in the above?

The bushel measure contains 2150.4 cubic inches. *Ans.* 49*b.* 3*pks.*

39. There is a cube 5ft. 4in. wide, 6ft high, and 12ft. 10in. long; how many bushels will it hold? *Ans.* 330.

365. *To measure a* CYLINDER.

DEF. A CYLINDER is a ROUND BODY, of equal size from end to end.

RULE. Multiply the area of ONE END by the LENGTH.

40. What is the solidity of a cylinder, whose ends are 12 inches in diameter, and length 120? *Ans.* 13571.712 *inches.*

41. A's half bushel measure is 13½ inches in diameter, and 7½ inches deep; is it too big or too little? if so, how much?

Ans. It is too little by 1.6568 *cubic in.*

366. *To measure a* PRISM.

DEF. A PRISM is a body with TWO EQUAL ENDS, and is either SQUARE, TRIANGULAR, OR POLYGONAL.

RULE. Multiply the area of ONE END by the LENGTH.

42. Required the solidity of a triangular prism, whose sides are 12 inches, and length 72.

The tabular number is 433013, Art. 309, therefore, *Ans.* 4490.3*in.* +.

43. What is the solidity of a pentagonal prism, whose sides are 18 inches, and length 96? *Ans.* 53513.7 + *in.*

367. *To measure a* PYRAMID.

DEF. 1. SOLIDS which DECREASE *gradually* till they come to a POINT, are generally called PYRAMIDS. If the BASE be a *square*, it is called a SQUARE PYRAMID; if a *triangle*, a TRIANGULAR PYRAMID; if a *circle*, a CIRCULAR PYRAMID, OR CONE.

2. The POINT AT THE TOP of a pyramid is called the VERTEX.

RULE. Multiply the AREA OF THE BASE by ⅓ of the PERPENDICULAR HEIGHT.

44. Required the solidity of a square pyramid, each side of whose base is 20 inches, and perpendicular height 72. *Ans.* 9600*in.*

45. Each side of the base of a triangular pyramid is 24 inches, and its height 96; what is its solidity? *Ans.* 7981.29*in.*

46. What is the solidity of a cone whose height is 21 feet, and *feet diameter?* Ans.

368. *To measure the* FRUSTUM *of a pyramid.*

DEF. The FRUSTUM of a pyramid is what REMAINS after the TOP IS CUT OFF; as a common churn, a tapering square, triangular, or polygonal stick of timber.

If the frustum be *square, triangular,* or *polygonal,*

RULE. To the RECTANGLE of ONE side of EACH end, add $\frac{1}{3}$ of the square of THE DIFFERENCE OF THOSE SIDES; multiply this sum by the TABULAR NUMBER, and that product by the LENGTH, and it gives the solidity.

But if it be a *frustum of a* CONE

RULE. To the rectangle of the DIAMETERS, add $\frac{1}{3}$ of the square of THEIR DIFFERENCE; multiply this sum by .7854, and that product by the DEPTH, and it gives the solid contents.

47. The length of a triangular frustum of a pyramid, is 120 inches, the greater base is 18, and the lesser 12 on one side; how many solid inches does it contain? *Ans.* 11847.23569.

48. There is a churn, whose top diameter is 10 inches, bottom diameter 13, and depth 36; how many wine gallons will it hold?

The wine gallon contains 231 cubic inches. *Ans.* 16.2792.

49. How many solid inches in a vessel 5 inches across at the top, and 3 at the bottom, the depth being 12? *Ans.* 153.9384.

369. *To measure a* SPHERE *or* GLOBE.

DEF. A SPHERE or GLOBE is a ROUND SOLID like an ORANGE.

The AREA of a globe is 4 times as much as the area of a *circle* of the same diameter, and may therefore be found by this

RULE. Multiply the WHOLE CIRCUMFERENCE into the WHOLE DIAMETER. (See Art. 310.)

50. Harry's ball is 7 inches in circumference; how many inches of leather will it take to cover it? *Ans.* 15.597 +.

51. The diameter of the globe we inhabit is 7964.12 miles; required the area of its surface. *Ans.* 199262914.687415 +.

370. *To find the* SOLIDITY OF A GLOBE.

RULE. Multiply the AREA OF ITS SURFACE by $\frac{1}{6}$ of its diameter.

52. How many cubic miles in the globe we inhabit?

Ans. 264491629810.340353344.

53. Required the solidity of a globe, that may be turned from a cubic block, containing 1 solid foot. *Ans.* .5236.

371. Hence, *if the diameter of a globe be given, the* SOLIDITY *may be found by the following*

RULE. MULTIPLY THE CUBE OF THE DIAMETER BY .5236.

54. The diameter of Herschel is 35112 miles; required its solidity.

Ans. 126650377223786.7008 *miles.*

372. *To measure a* SEGMENT OF A GLOBE.

RULE. To THREE TIMES the square of the SEMIDIAMETER OF THE BASE, add the square of the HEIGHT, multiply THIS SUM by the height, and that PRODUCT by .5236.

55. The height of a segment of a globe is 9 inches, and the diameter of its base 24; what is its solidity? Ans. 2417.46 +*cubic inches*

373. *To measure a* SPHEROID.

DEF. A SPHEROID is a LONG SPHERE like an egg, only both its ends are equal.

RULE. Multiply the square of the SHORTEST DIAMETER by the LONGEST, and that product by .5236.

56. If the two diameters of a spheroid be 20 and 30, what is its solidity? *Ans.* 6283.2.

374. *To measure a* SEGMENT *of a* SPHEROID.*

RULE. TO FOUR TIMES THE SQUARE of the MIDDLE DIAMETER, add the SQUARE of the BASE DIAMETER; multiply this sum by .7854, and that product by $\frac{1}{6}$ of the HEIGHT.

57. If the base diameter be 36 inches, diameter at the middle of the height 30, and height 20, what is the solidity? *Ans.* 12689.55+.

375. *To measure the* MIDDLE ZONE *or* FRUSTUM OF A GLOBE *or* SPHEROID.

RULE. TO TWICE THE SQUARE of the MIDDLE DIAMETER, add the SQUARE of the END DIAMETER; multiply this sum by .7854, and that product by ONE THIRD of the length.

58. If the middle diameter of a zone be 20 inches, end diameters each 16, and length 12, what is its solidity? *Ans.* 3317.529+.

59. If the middle and end diameters of the middle frustum of a spheroid be 40 and 30, and length 50, what is its solidity? *Ans.* 53454.3+.

376. *To measure a* SOLID RING.

RULE. To the CIRCUMFERENCE OF THE RING multiplied by .31831, (=1÷3.1416) add the INTERNAL DIAMETER; multiply this sum by 3.1416, and the product will be the length of a CYLINDER EQUAL TO THE RING of the same base.

60. If an iron ring be 12 inches in girth, and its internal diameter 20 inches, what is its solidity? *Ans.* 847.97.

377. *To find the solidity of* ANY IRREGULAR BODY.

RULE. IMMERSE IT in a vessel partly full of water; then the MEASURE OF THE DISTANCE it raises the fluid will be the SOLIDITY REQUIRED.

61. How many cubic inches in a brush heap as big as a house 20 feet square, and 15 high; supposing that the brush, thrown into a cistern of water, of the same horizontal size, raises the fluid 5 feet? *Ans.* 34560000.

378. There are FIVE SOLIDS contained under *equal regular sides*, which by way of distinction, are called THE FIVE REGULAR BODIES.

These with their *names, number of sides*, SOLIDITIES *and* SUPERFICIES, (*the sides being* 1) may be seen in the following

TABLE.

SIDES.	NAMES.	SOLIDITIES.	SUPERFICIES.
4	Tetrædron	0.11785.	1.73205
6	Hexædron or CUBE	1.....	6.....
8	Octædron	0.4714.	3.464..
12	Dodecædron	7.663119	20.6457.
20	Eicosiedron	2.181695	8.66025

* *Made by* a plane parallel to the greatest diameter.

The solidity of any of the foregoing bodies may evidently be found by multiplying the area of its side by the CIRCULAR NUMBER.

CASK GUAGING.

379. CASKS in general may be reduced to CYLINDERS, by adding ⅔ to the head diameter TWO THIRDS, or, (if the staves be but little curving,) SIX TENTHS of the difference between the head and bung diameters.

The wine gallon contains 231, and the beer gallon 282 cubic inches: therefore $\frac{1}{231} = .0034$, and $\frac{1}{282} = .0035$.

Hence, for MEASURING OR GUAGING CASKS, we have this

RULE. Multiply the SQUARE of the MEAN DIAMETER by the LENGTH and THIS PRODUCT by .0034 for WINE, and by .0035 for BEER GALLONS. Or,

With any straight rod, take the DIAGONAL OF THE CASK, from the centre of the bung hole, WITH WAYS; then multiply the CUBE of the DIAGONAL IN INCHES by .002328 for ALE; and by .002772 for WINE GALLONS.

62. How many wine gallons in a cask whose bung diameter is 33 inches, head diameter 25, and length 47? *Ans.* 147.+

63. How many gallons in a cask whose diagonal is 36 inches?
Ans. 103.949669 Ale, or 126.90432 Wine gallons.

380. To find the distance a SPECTATOR CAN SEE at any given elevation above the earth's surface.

RULE. Multiply the SUM of the EARTH'S DIAMETER (=42056462ft.) and the ELEVATION OF THE EYE, by the ELEVATION; and the SQUARE ROOT of the product, will be the DISTANCE WHICH CAN BE SEEN IN MILES.

Hence we may find the distance which objects may be seen (THE HEIGHTS BEING GIVEN,) by *working for the* ELEVATION OF THE EYE *and the* ELEVATION OF THE OBJECT *separately; the* SUM OF THE RESULTS *will be the* ANSWER.

64. How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated 5 feet above the surface? *Ans.* 91.999 *miles.*

381. To estimate the HEIGHT OF OBJECTS on level ground, or at sea, having only the distance given,

RULE. From the GIVEN DISTANCE, take the DISTANCE WHICH THE ELEVATION OF YOUR EYE WILL GIVE; and divide the SQUARE of the remainder IN FEET by 42056462.

65. Being on my return from a foreign voyage, and finding by my reckoning I was about 6½ leagues from Boston light-house, it being in the dusk of the evening, I descried the lamp in the horizon, at which time my eye was elevated 10 feet above the surface of the water: Now, supposing my reckoning to be true, what is the height of the light-house above the water? *Ans.* 161½ *feet.*

ALGEBRA.*

382. ALGEBRA IS A METHOD OF COMPUTING BY LETTERS AND OTHER SYMBOLS.

It is esteemed, by the author, as of more scientific value than any other branch of the mathematics. It is indeed the ROOT of all the others; and may justly be styled the RULE FOR MAKING RULES.

383. The author would gladly treat of this analytic art at some length, but the limits of his work will not admit. Suffice it to say, that the method of proceeding, is, to translate the quantities into letters, &c., and proceed with them as though they were the *quantities themselves*, until the unknown quantity is found to be equal to some KNOWN ONES.

383 *a*. In ALGEBRA, when two or more letters are written or printed TOGETHER, they are understood to be MULTIPLIED.

Thus ab denotes that a is to be multiplied by b.

NOTE. For the known quantities, it is common, to put the first letters of the alphabet, as a, b, *&c.*; and for the unknown, the last letters, as x, y, z, &c.

Ex. 1. A stake is above water 10 feet, under water 6 feet, and in the ground *half its length* wanting 4 feet; how far is it in the ground?

Put $10 = a$, $6 = b$, and $4 = c$; and let $x =$ the distance required.

Then, by the problem, $x + c = a + b$

And, $x = a + b - c = 10 + 6 - 4 = 12$ *feet, Ans.*

383 *b*. The expression, $x + c = a + b$, is called an EQUATION; and all BEFORE and AFTER the sign of EQUALITY, are called ITS SIDES.

384. To transpose any letter, is to change it from one side to the other; and when this is done, the SIGN of that letter *must also be changed.*

Thus, in the above, c is transposed, and its sign changed from $+$ to $-$

Ex. 2. Find two numbers, whose sum shall be 40, and difference 16.

Put $40 = a$, and $16 = b$; and let $x =$ the lesser number. Then will $x + b =$ the greater. And, by the problem, $x + x + b = a$

Transposing b, &c. $2x = a - b$. Hence $x = \frac{a-b}{2} = 12$, = the lesser number; then $12 + 16 = 28$, = the greater.

TRIGONOMETRICAL, AND OTHER SURVEYING.

385. SURVEYING IS THE ART OF MEASURING LAND.

386. TRIGONOMETRY is the art of measuring TRIANGLES.

387. The PERIPHERY of every circle is supposed to be divided into 360 equal parts, called DEGREES; and each degree into 60 other equal parts, called *minutes*, &c.

* Those who would wish to pursue this study, cannot perhaps be referred to better *elementary* treatise, than Day's Algebra.

Hence a quadrant, or quarter of a circle, contains 90 degrees, and a semicir cle 180.

388. The difference of any arch from 90° is called its COMPLEMENT; and its difference from 180, ITS SUPPLEMENT.

389. The SINE of any arch is a RIGHT LINE* drawn from ONE EXTREMITY of it perpendicular to a diameter passing through the OTHER EXTREMITY.

390. The VERSED SINE *is that part of the diameter* between the SINE and the PERIPHERY.†

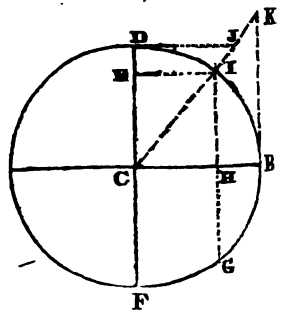
391. The COSINE‡ is that part of the diameter, between the SINE and the CENTRE.

392. The TANGENT of an arch is a RIGHT LINE, touching the ARCH in *one extremity*, and produced till it meets a line drawn FROM THE CENTRE, through the *other extremity*.

393. The SECANT is a line drawn from the CENTRE to the EXTREMITY OF THE TANGENT.

394. The COTANGENT and COSECANT, are the TANGENT and SECANT *of the* COMPLEMENT *of the arch*.

RECAPITULATION. Let B I be a given arch; then is I H its SINE, H B its VERSED SINE, C H (= E I) its COSINE, B K its TANGENT, C K its SECANT, D J its COTANGENT, and C J its COSECANT.



395. TO CALCULATE A TABLE OF NATURAL SINES.

The semicircumference of the circle whose radius is 1, is 3.1; 1592653589793, &c; it will be as the number of degrees or minutes in the whole semicircle : is to the degrees or minutes in the arch proposed : : so is 3.14159, &c. : to the length of said arch; which let be denoted by a; then

Its sine will be expressed by $a - \frac{a^3}{2.3} + \frac{a^5}{2.3.4.5} - \frac{a^7}{2.3.4.5.6.7}$ &c.

And its cosine by $1 - \frac{a^2}{2} + \frac{a^4}{2.3.4} - \frac{a^6}{2.3.4.5.6} + \frac{a^8}{2.3.4.5.6.7.8}$ &c.

EX. 1. Required the sine and cosine of 1 minute.

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*wise *straight line*. † Circumference.
equal to the SINE of the *complement* of the arch; the co being a cos at word.
d is here used as the sign of multiplication; that is, instead of ×

The number of minutes in 180° being 10800, it will be first, as 10800 : 1 :: 3.14159265, &c. : .0002908882306605 = the length of an arc of one minute.
Then, in this case, a = .0002908882
and $a^3 \div (2 \times 3)$ = .000000000004, &c.
Their difference is .0002908882, the SINE required.

Also, from 1., take $a^2 \div 2$ = 0.0000000423079, &c.
And it leaves .9999999577, the COSINE.

2. Required the sine and cosine of 5 minutes.

Ans., the sine is .08715574, & *cosine*, .9961947.

396. After the same manner, the sine and cosine of any other arch or arc may be found. But when the sine is found, the cosine may be more easily obtained by the following

RULE. Add the sine to ONE and subtract it from ONE; multiply the SUM by the REMAINDER, and the SQUARE ROOT of the product will be the COSINE.

397. The SINE of any arch divided by the COSINE, gives the NATURAL TANGENT of that arch.

398. SQUARE THE TANGENT, and ADD ONE TO THE SQUARE, and the SQUARE ROOT of that SUM, will be the NATURAL SECANT.

399. TO COMPUTE THE TANGENTS AND SECANTS.

The SINES and COSINES being known, or found as before, the TANGENTS and SECANTS will be easily obtained, from the principle of SIMILAR TRIANGLES.

Thus, in the figure, of the arc IB, IH is the SINE, CH or EI, the cosine, BK the tangent, CK the secant, DJ the cotangent, and CJ the cosecant, the radius being CA or CB. First then,

CH : HI :: CB : BK; whence the TANGENT is known, being a fourth proportional to the COSINE, SINE, and RADIUS.

2d. CH : CI :: CB : CK; whence the SECANT is known, being a third proportional to the COSINE and RADIUS.

3d. IH : HC :: CD : DJ; whence the COTANGENT is known, being a fourth proportional to the SINE, COSINE, and RADIUS.

4th. IH : IC :: CD : CJ; whence the COSECANT is known, being a third proportional to the SINE and RADIUS.

The following is an EPITOME OF THE TABLE OF NATURAL SINES, &c.

TABLE I.

D.	M.	Sine.	Cosine.	D.	M.
0°	30′	.0087, 2653	.9999, 6192	89°	30′
1	30	.0261, 7694	.9996, 5732	88	30
2	30	.0436, 1938	.9990, 4822	87	30
3	30	.0610, 4853	.9981, 3479	86	30
4	30	.0784, 5909	.9969, 1733	85	30
5	30	.0958, 4575	.9953, 9619	84	30
6	30	.1132, 0321	.9935, 7185	83	30
7	30	.1306, 2619	.9914, 4486	82	30
8	30	.1487, 0941	.9890, 1586	81	30
9	30	.1650, 4760	.9862, 8560	80	30
10	30	.1822, 3552	.9832, 5490	79	30
11	30	.1993, 6793	.9799, 2470	78	30
12	30	.2164, 3961	.9762, 9600	77	30
13	30	.2334, 4536	.9723, 6992	76	30
14	30	.2503, 8000	.9681, 4764	75	30
15	30	.2672, 3837	.9636, 3045	74	30
16	30	.2840, 1534	.9588, 1973	73	30
17	30	.3007, 0579	.9537, 1695	72	30
18	30	.3173 0465	.9483, 2365	71	30
19	30	.3338, 0685	.9426, 4149	70	30
20	30	.3502, 0738	.9366, 7218	69	30
21	30	.3665, 0122	.9304, 1756	68	30
22	30	.3826, 8343	.9238, 7953	67	30
23	30	.3987, 4906	.9170, 6007	66	30
24	30	.4146, 9324	.9099, 6127	65	30
25	30	.4305, 1109	.9025, 8528	64	30
26	30	.4461, 9781	.8949, 3436	63	30
27	30	.4617, 4861	.8870, 0083	62	30
28	30	.4771, 5876	.8788, 1711	61	30
29	30	.4924, 2356	.8703, 5569	60	30
30	30	.5075, 3836	.8616, 2916	59	30
31	30	.5224, 9856	.8526, 4016	58	30
32	30	.5372, 9960	.8433, 9144	57	30
33	30	.5519, 3698	.8338, 8582	56	30
34	30	.5664, 0623	.8241, 2618	55	30
35	30	.5807, 0295	.8141, 1551	54	30
36	30	.5948, 2278	.8038, 5686	53	30
37	30	.6087, 6142	.7933, 5334	52	30
38	30	.6225, 1463	.7826, 0185	51	30
39	30	.6360, 7822	.7716, 2458	50	30
40	30	.6494, 4804	.7604, 0596	49	30
41	30	.6626, 2004	.7489, 5572	48	30
42	30	.6755, 9020	.7372, 7733	47	30
43	30	.6883, 5457	.7253, 7437	46	30
44	30	.7009, 0926	.7132, 5044	45	30
		Cosine.	Sine.	D.	M.

	D.	Sine.	Cosine.		
291	0°	.0000, 0000	1.0000, 0000	90°	0
291	1	.0174, 5240	.9998, 4769	89	2
291	2	.0348, 9949	.9993, 9082	88	8
291	3	.0523, 3595	.9986, 2953	87	13
291	4	.0697, 5647	.9975, 6405	86	18
290	5	.0871, 5574	.9961, 9469	85	23
289	6	.1045, 2846	.9945, 2189	84	30
289	7	.1218, 6934	.9925, 4615	83	33
288	8	.1391, 7310	.9902, 6806	82	38
287	9	.1564, 3446	.9876, 8834	81	43
286	10	.1736, 4817	.9848, 0775	80	48
285	11	.1908, 0899	.9816, 2718	79	53
284	12	.2079, 1169	.9781, 4760	78	58
283	13	.2249, 5105	.9743, 7006	77	63
282	14	.2419, 2189	.9702, 9572	76	68
281	15	.2588, 1904	.9659, 2582	75	73
279	16	.2756, 3735	.9612, 6169	74	78
278	17	.2923, 7170	.9563, 0475	73	83
276	18	.3090, 1699	.9510, 5651	72	88
275	19	.3255, 6815	.9455, 1857	71	93
273	20	.3420, 2014	.9396, 9262	70	97
271	21	.3583, 6794	.9335, 8042	69	102
269	22	.3746, 0659	.9271, 8385	68	107
267	23	.3907, 3112	.9205, 0485	67	112
265	24	.4067, 3664	.9135, 4545	66	116
263	25	.4226, 1826	.9063, 0778	65	121
261	26	.4383, 7114	.8987, 8404	64	126
258	27	.4539, 9049	.8910, 0652	63	130
256	28	.4694, 7156	.8829, 4759	62	135
254	29	.4848, 0962	.8746, 1970	61	139
251	30	.5000, 0000	.8660, 2540	60	144
248	31	.5150, 3807	.8571, 6730	59	148
246	32	.5299, 1926	.8480, 4809	58	152
243	33	.5446, 3903	.8386, 7056	57	157
240	34	.5591, 9290	.8290, 3757	56	161
237	35	.5735, 7643	.8191, 5204	55	165
234	36	.5877, 8525	.8090, 1699	54	169
231	37	.6018, 1502	.7986, 3551	53	173
228	38	.6156, 6147	.7880, 1075	52	177
225	39	.6293, 2039	.7771, 4596	51	181
221	40	.6427, 8760	.7660, 4444	50	185
218	41	.6560, 5902	.7547, 0958	49	189
215	42	.6691, 3060	.7431, 4482	48	193
211	43	.6819, 9836	.7313, 5370	47	197
208	44	.6946, 5837	.7193, 3980	46	201
204	45	.7071, 0678	.7071, 0678	45	204
		Cosine.	Sine.	D.	

TABLE III.

TANGENTS from 1° to 45°.

D.				D.			
0	.000000	17449	0.1	23	.424475	20601	2.5
1	.017455	17460	0.1	24	.445229	20915	2.7
2	.034921	17477	0.2	25	.466308	21251	2.9
3	.052408	17503	0.3				
4	.069927	17539	0.4	26	.487733	21609	3.1
5	.087489	17588	0.5	27	.509525	21988	3.3
				28	.531709	22392	3.5
6	.105104	17648	0.5	29	.554309	22821	3.7
7	.122785	17719	0.6	30	.577350	23276	3.9
8	.140541	17799	0.7				
9	.158384	17893	0.8	31	.600861	23759	4.2
10	.176327	17997	0.9	32	.624869	24274	4.4
				33	.649408	24819	4.7
11	.194880	18114	1.0	34	.674509	25399	5.
12	.212557	18244	1.1	35	.700208	26017	5.3
13	.230868	18386	1.2				
14	.249328	18541	1.3	36	.726543	26674	5.6
15	.267949	18708	1.5	37	.753554	27371	6.
				38	.781286	28115	6.4
16	.286745	18891	1.6	39	.809784	28907	6.8
17	.305731	19088	1.7	40	.839100	29751	7.3
18	.324920	19299	1.8				
19	.344328	19525	2.0	41	.869287	30651	7.8
20	.363970	19768	2.1	42	.900404	31613	8.3
				43	.932515	32642	8.9
21	.383864	20028	2.2	44	.965689	33743	9.5
22	.404026	20305	2.4	45	1.000000	00000	0.0

TABLE IV.

TANGENTS from 45° to 90°.

D.		D.			D.
46	1.1355303	61	1.8040477	4.0107809	76
47	1.0723687	62	1.8807264	4.3314789	77
48	1.1106125	63	1.9626105	4.7046301	78
49	1.1503684	64	2.0503038	5.1445540	79
50	1.1917536	65	2.1445069	5.6712818	80
51	1.2348971	66	2.2460367	6.3137515	81
52	1.2799416	67	2.3558524	7.1153697	82
53	1.3270448	68	2.4750868	8.1443464	83
54	1.3763819	69	2.6050891	9.5143644	84
55	1.4281480	70	2.7474774	11.430052	85
56	1.4825609	71	2.9042109	14.300652	86
57	1.5398649	72	3.0776835	19.081136	87
58	1.6003345	73	3.2708526	28.636253	88
59	1.6642794	74	3.4874144	57.289962	89
60	1.7320508	75	3.7320585	Infinite.	90

TABLE V.

By which to find the sine of *minutes*.

D.			D.		
0	29089.57	0.0365	45	20569.37	3.0166
1	29085.12	0.1105	46	20207.17	3.0688
2	29071.83	0.1843	47	19839.00	3.1188
3	29049.67	0.2577	48	19464.73	3.1687
4	29018.68	0.3315	49	19084.43	3.2165
5	28978.87	0.4055	50	18698.40	3.2644
6	28930.20	0.4788	51	18306.62	3.3105
7	28872.73	0.5521	52	17909.33	3.3565
8	28806.47	0.6255	53	17506.53	3.4010
9	28731.42	0.6983	54	17098.38	3.4437
10	28647.63	0.7710	55	16685.05	3.4861
11	28555.10	0.8433	56	16266.68	3.5282
12	28453.87	0.9155	57	15843.32	3.5683
13	28344.00	0.9877	58	15415.12	3.6072
14	28225.48	1.0592	59	14982.23	3.6454
15	28098.35	1.1305	60	14544.77	3.6822
16	27962.68	1.2015	61	14102.92	3.7183
17	27818.45	1.2505	62	13656.72	3.7527
18	27665.80	1.3422	63	13206.40	3.7866
19	27504.68	1.4115	64	12752.03	3.8187
20	27335.27	1.4822	65	12293.82	3.8505
21	27157.45	1.5505	66	11831.80	3.8800
22	26971.38	1.6193	67	11366.20	3.9088
23	26777.07	1.6866	68	10897.12	3.9361
24	26574.63	1.7543	69	10424.73	3.9621
25	26364.07	1.8211	70	9949.22	3.9883
26	26145.55	1.8883	71	9470.63	4.0121
27	25919.02	1.9538	72	8989.20	4.0355
28	25684.57	2.0188	73	8514.97	4.0566
29	25442.30	2.0883	74	8018.14	4.0771
30	25192.28	2.1471	75	7528.97	4.0966
31	24934.62	2.2105	76	7037.43	4.1143
32	24669.32	2.2727	77	6543.70	4.1300
33	24396.55	2.3350	78	6048.03	4.1454
34	24116.32	2.3961	79	5550.52	4.1594
35	23828.77	2.4566	80	5051.35	4.1727
36	23533.92	2.5161	81	4550.60	4.1844
37	23231.92	2.5760	82	4048.52	4.1950
38	22922.87	2.6333	83	3545.10	4.2033
39	22606.85	2.6916	84	3040.67	4.2111
40	22283.90	2.7477	85	2535.33	4.2176
41	21954.17	2.8033	86	2029.13	4.2221
42	21617.73	2.8576	87	1522.45	4.2272
43	21274.72	2.9116	88	1015.22	4.2294
44	20925.25	2.9650	89	507.68	4.2304

TABLE VI.

NATURAL SINES for *degrees* and *minutes*.

M.	Sine. 0°	Cosine.	
1	.00029	UNITY,	59
6	.00175	*nearly.*	54
15	.00463	.99999	45
20	.00582	.99998	40
25	.00727	.99997	35
35	.01018	.99995	25
40	.01164	.99993	20
45	.01309	.99991	15
50	.01454	.99989	10
55	.01600	.99987	5
	89°	89°	
	1°	1°	
15	.02181	.99976	45
20	.02327	.99973	40
40	.02908	.99958	20
45	.03054	.99953	15
	88°	88°	
	2°	2°	
15	.03926	.99923	45
20	.04071	.99917	40
40	.04652	.99892	20
45	.04798	.99885	15
	87°	87°	
	3°	3°	
15	.05669	.99839	45
20	.05814	.99831	40
40	.06395	.99795	20
45	.06540	.99786	15
	86°	86°	
	4°	4°	
15	.07411	.99725	45
20	.07556	.99714	40
40	.08136	.99668	20
45	.08281	.99657	15
	85°	85°	
	5°	5°	
15	.09150	.99580	45
20	.09295	.99567	40
40	.09874	.99511	20
45	.10019	.99497	15
	84°	84°	
	6°	6°	
15	.10887	.99406	45
20	.11031	.99390	40
40	.11609	.99324	20
45	.11754	.99307	15
	83°	83°	
	Cosine.	Sine.	M.

M.	Sine.	Cosine.	
	7°	7°	
15	.12620	.99200	45
20	.12764	.99182	40
40	.13341	.99106	20
45	.13485	.99087	15
	82°	82°	
	8°	8°	
15	.14349	.98965	45
20	.14493	.98944	40
40	.15069	.98858	20
45	.15212	.98836	15
	81°	81°	
	9°	9°	
15	.16074	.98700	45
20	.16218	.98676	40
40	.16792	.98580	20
45	.16935	.98556	15
	80°	80°	
	10°	10°	
15	.17794	.98404	45
20	.17937	.98378	40
40	.18509	.98079	20
45	.18652	.98245	15
	79°	79°	
	11°	11°	
15	.19509	.98079	45
20	.19652	.98050	40
40	.20222	.97934	20
45	.20364	.97905	15
	78°	78°	
	12°	12°	
15	.21218	.98079	45
20	.21360	.97692	40
40	.21928	.97566	20
45	.22070	.97534	15
	77°	77°	
	13°	13°	
15	.22920	.97338	45
20	.23062	.97304	40
40	.23627	.97169	20
45	.23779	.97134	15
	76°	76°	
	Cosine.	Sine.	M.

TABLE VI.

M.	Sine.	Cosine.		M.	Sine.	Cosine.	
	14°	14°			21°	21°	
15	.24615	.96923	45	15	.36244	.93201	45
20	.24756	.96887	40	20	.36379	.93148	40
40	.25320	.96742	20	40	.36921	.92935	20
45	.25460	.96705	15	45	.37056	.92881	15
	75°	75°			68°	68°	
	15°	15°			22°	22°	
15	.26303	.96479	45	15	.37865	.92554	45
20	.24756	.96887	40	20	.37999	.92499	40
40	.27004	.96285	20	40	.38538	.92276	20
45	.27144	.96246	15	45	.38671	.92220	15
	74°	74°			67°	67°	
	16°	16°			23°	23°	
15	.27985	.96005	45	15	.39474	.91879	45
20	.28123	.95964	40	20	.39608	.91822	40
40	.28680	.95799	20	40	.40141	.91590	20
45	.28820	.95757	15	45	.40275	.91531	15
	73°	73°			66°	66°	
	17°	17°			24°	24°	
15	.29654	.95502	45	15	.41072	.91176	45
20	.29793	.94459	40	20	.41204	.91116	40
40	.30348	.95285	20	40	.41734	.90875	20
45	.30486	.95240	15	45	.41866	.90814	15
	72°	72°			65°	65°	
	18°	18°			25°	25°	
15	.31316	.94970	45	15	.42657	.90446	45
20	.31454	.94924	40	20	.42788	.90383	40
40	.32006	.94740	20	40	.43313	.90133	20
45	.32144	.94693	15	45	.43445	.90070	15
	71°	71°			64°	64°	
	19°	19°			26°	26°	
15	.32969	.94408	45	15	.44227	.89687	45
20	.33106	.94361	40	20	.44359	.89623	40
40	.33635	.94167	20	40	.44880	.89363	20
45	.33792	.94118	15	45	.45010	.89298	15
	70°	70°			63°	63°	
	20°	20°			27°	27°	
15	.34612	.93819	45	15	.45787	.88902	45
20	.34747	.93769	40	20	.45917	.88835	40
40	.35297	.93565	20	40	.46433	.88566	20
45	.35429	.93514	15	45	.46561	.88499	15
	69°	69°			62°	62°	
	Cosine.	Sine.	M.		Cosine.	Sine.	M.

TABLE VI.

M.	Sine.	Cosine.	
	28°	28°	
15	.47332	.88089	45
20	.47460	.88020	40
40	.47971	.87743	20
45	.48150	.87645	15
	61°	61°	
	29°	29°	
15	.48862	.87250	45
20	.48987	.87178	40
40	.49495	.86892	20
45	.49622	.86820	15
	60°	60°	
	30°	30°	
15	.50377	.86384	45
20	.50503	.86295	40
40	.51004	.86015	20
45	.51129	.85941	15
	59°	59°	
	31°	31°	
15	.51877	.85491	45
20	.52002	.85416	40
40	.52498	.85112	20
45	.52621	.85035	15
	58°	58°	
	32°	32°	
15	.53361	.84573	45
20	.53484	.84495	40
40	.53975	.84182	20
45	.54097	.84104	15
	57°	57°	
	33°	33°	
15	.54829	.83629	45
20	.54959	.83549	40
40	.55436	.83228	20
45	.55557	.83147	15
	56°	56°	
	34°	34°	
15	.56280	.82659	45
20	.56401	.82577	40
40	.56880	.82248	20
45	.57000	.82165	15
	55°	55°	
	Cosine.	Sine.	M.

M.	Sine.	Cosine.	
	35°	35°	
15	.57715	.81664	45
20	.57833	.81580	40
40	.58307	.81242	20
45	.58425	.81157	15
	54°	54°	
	36°	36°	
15	.59131	.80644	45
20	.59248	.80558	40
40	.59716	.80212	20
45	.59832	.80125	15
	53°	53°	
	37°	37°	
15	.60529	.79600	45
20	.60645	.79512	40
40	.61107	.79158	20
45	.61222	.79069	15
	52°	52°	
	38°	38°	
15	.61909	.78532	45
20	.62024	.78442	40
40	.62479	.78079	20
45	.62592	.77988	15
	51°	51°	
	39°	39°	
15	.63271	.77439	45
20	.63383	.77347	40
40	.63832	.76977	20
45	.63944	.76884	15
	50°	50°	
	40°	40°	
15	.64612	.76323	45
20	.64723	.76229	40
40	.65166	.75851	20
45	.65276	.75756	15
	49°	49°	
	41°	41°	
15	.65935	.75184	45
20	.66044	.75088	40
40	.66480	.74703	20
45	.66588	.74606	15
	48°	48°	
	Cosine.	Sine.	M.

TABLE VI.

M.	Sine.	Cosine.		M.	Sine.	Cosine.	
	42°	42°		40	.69046	.72337	20
15	.67237	.72837	45	45	.69151	.72236	15
20	.67344	.73924	40		46°	46°	
40	.67773	.73531	20		44°	44°	
45	.67880	.73432	15	15	.69779	.71630	45
	47°	47°		20	.69883	.71529	40
	43°	43°		40	.70298	.71121	20
15	.68518	72837	45	45	.70401	.71019	15
20	.68624	72737	40		45°	45°	
	Cosine.	Sine.	M.		Cosine.	Sine.	M.

400. TABLE II, contains sines for degrees only; but by the number on the outside of that table, may be found the sine for minutes to 4 or 5 places, sufficiently exact.

RULE. Take SIX PLACES of figures out of said table, for the SINE OF THE DEGREES; then multiply the given minutes into the number on the OUTSIDE COLUMN, that stands against THOSE DEGREES; and add THE PRODUCT to THAT SINE, and it will give the DEGREES AND MINUTES required.

EXAMPLE 1. Find the sine of 25° 19′. *Ans.* .427615.
..... 2. .... of 39° 42′. *Ans.* .63877.
..... 3. .... of 42° 39′. *Ans.* .677515.

401. *To find the sine of minutes to* 7 PLACES *by* TABLE V.

RULE. For all minutes below 25, take two figures of decimals of the number in the third column, and add an unit to the second figure; then multiply the MINUTES into *that number**, and subtract the product from the number in the second column; multiply the remainder by the minutes, and add the INTEGERS to the *sine of the degrees* taken out of table 2d, and it will give the SINE REQUIRED.

NOTE 1. For all minutes from 25 to 55, take 3 decimals in the 3d column, (without any addition to the 3d figure,) and proceed as before.

2. For all minutes from 55 to 59, take *all the figures* in the 3d column.

EXAMPLE 1. Find the sine of 23° 17′. *Ans.* .3952837.
..... 2. .... of 35° 49′. *Ans.* .5851935.
..... 3. .... of 52° 51′. *Ans.* .7970572.
..... 4. .... of 43° 56′. *Ans.* .6938209.

402. To find the *degrees and minutes* answering to *any sine*, GIVEN *or* FOUND BY PROCESS.

RULE. Find the nearest NEXT LESS in table 2d, and subtract it from the GIVEN SINE; divide the remainder by the number in the outer column against SAID NEXT LESS SINE, and the quotient will be the MINUTES.

NOTE. The given sine must contain just 6 places.

* Always retaining two decimals.

Ex. What are the degrees and minutes answering to .727845? —— .523678? —— .767520? *Ans. to last*, 45° 2′.

402. To find the degrees and minutes answering to any GIVEN TANGENT (which must always contain 6 places.)

RULE. Subtract the tangent NEXT LESS found in the table, from the GIVEN TANGENT, multiply the REMAINDER by 60; divide this product by the number in the THIRD COLUMN, against *said next less tangent, and the quotient will be the* MINUTES.

404. The LEAST LEG of any RIGHT ANGLED TRIANGLE, divided by the GREATEST, gives the TANGENT of the LEAST ANGLE.*

404 *a.* The GREATEST LEG, divided by the SINE OF THE GREATEST ACUTE ANGLE, gives the HYPOTENUSE; and the LEAST LEG, divided by the SINE OF THE LEAST ANGLE, gives the SAME.

405. EITHER LEG divided by the HYPOTENUSE, gives the SINE OF THE ANGLE OPPOSITE THAT LEG.

406. IN EVERY PLANE TRIANGLE, there are SIX THINGS, namely, THREE SIDES, and THREE ANGLES. THREE of these must be given to find the OTHERS, and among THESE THREE, there must be *at least* ONE SIDE.

406 *a.* IN ANY TRIANGLE, the SUM OF ALL THE ANGLES is equal to TWO RIGHT ANGLES, or 180°.

407. THE SIDES OF TRIANGLES are PROPORTIONAL to each other, as the SINES of their OPPOSITE ANGLES.

Ex. Given two sides = 24 and 30, and the angle opposite the former = 53° 30′, to find the other side and angles.

Ans. the other side is 22.95+, *and angles* 76° 15′ and 50° 15′.

408. When TWO SIDES and their contained angle, are given to find the remaining side and angle,

RULE. As the SUM of the TWO GIVEN SIDES, is to THEIR DIFFERENCE; so is the TANGENT OF HALF THE SUM OF THEIR OPPOSITE ANGLES to the tangent of HALF THE DIFFERENCE of those angles.

Then if we add the half difference of the angles, so found, to their half sum, it will give the GREATER ANGLE; and subtracting the same, will leave the LESSER ANGLE.

Ex. Given two sides = 64 and 47, and contained angle 31° 8′, to find the other side and angles.

Ans. the other side is 34, *and angles* 103° 15′, & 45° 37′.

409. Two sides, and the included angle of any triangle, being given to find the area.

RULE. Multiply the HALF PRODUCT of the TWO GIVEN SIDES by the SINE OF THE INCLUDED ANGLE

410. The area, contained angle, and one side of any triangle, given to find the adjoining side, including the angle.

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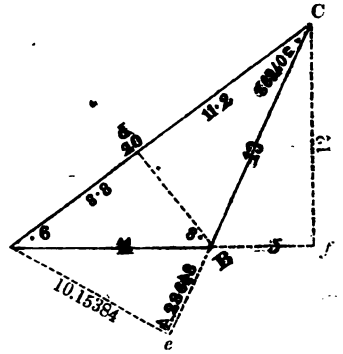
* See Art. 392, and the subsequent figure.

RULE. Multiply the SINE OF THE INCLUDED ANGLE into the GIVEN SIDE, and BY THE PRODUCT divide the DOUBLE AREA.

411. The DOUBLE AREA of any triangle *divided by the* PRODUCT *of any* TWO OF ITS SIDES, gives the SINE OF THE ANGLE BETWEEN THOSE SIDES.



412. Draw AB = 11, AC = 20, and BC = 13; then as AC (= 20) to AB + BC (24) : : so BC — AB (2) : to the diff. of A*d* and *d*C = 2.4. (Art. 349.) The half diff. 1.2, added to ½ AC = 10, gives *d*C = 11.2; and taken from ½ AC (= 10) leaves A*d* = 8.8. Then $\sqrt{11^2 - 8.8^2} = 6.6 = B*d*$.

412 *a*. To find the angle BAC, divide B*d* (6.6) by BA (11), and it gives .6, the natural sine of that angle = 36° 52′*.

412 *b*. The angle AB*d* = the angle AC*f*, because the angle BAD taken from 90° leaves both the one and the other, whose natural sine is .8 = 53° 8′.

Hence .8 × 11 (AB) gives 8.8 (A*d*), and .8 × 20 = 16 (A*f*); also 16—11 = 5 (B*f*), and .6 × 20 = 12 = C.

412 *c*. Multiply the SINE of the angle BCA (.507692) into AC (20), and it gives A*e* = 10.15384.

412 *d*. Multiply AC (20) into B*d* (6.6), and it gives the DOUBLE AREA of the triangle ABC = 132; which divided by *either* side of the triangle, gives the PERPENDICULAR. Raise the perpendicular *on that side*, and it shall include the SINGLE AREA. Thus,

1. AC = 20) 132 (6.6, B*d*.
2. AB = 11) 132 (12, C*f*.
3. BC = 13) 132 (10.15384, A*e*.

The supplement of the angle ABC is .923077 = 67° 23′, which taken from 180°, leaves 112° 37′, the angle ABC. Therefore,

1st. Given the angle BAC, the side AC, and the AREA; required the side AB.

412 *e*. RULE. *Divide the* DOUBLE AREA *by the given side*, and it gives the PERP. *opposite the given angle* (= 6.6); then as .6 : 6.6 : : 1 : 11 = AB.

2nd. Given the same angle, the area, and the side AB; required the side AC.

412 *f*. RULE. *The double area divided by the given side*

* To find the angle ABC, 13) 6.6 (= .507692 = 30° 30′ 32″.

AB, *gives the* PERP. fC (= 12): then as .6 : 12 : : 1 : 20 = AC, the side required.

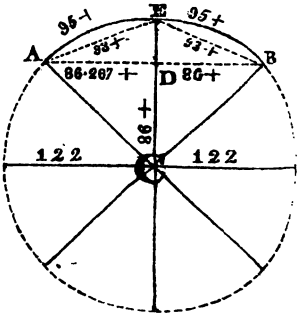
3d. Given the angle BCA, the area, and the side BC; required the side AC.

412 *g. The double area divided by the side* BC *gives the* PERPENDICULAR *opposite the given angle,* (= *Ae*) = 10.15384; as .507692: 10.15384 : : 1 : 20, AC.

413. IN PRACTICAL SURVEYING, the angle ABe would be found by the compass; but it may be found in the figure.

THUS:—Take the sum of the angles BCA & BAC (= 67° 22′) from 90°, leaves BA*e* = 22° 38′; this taken again from 90°, leaves the angle CB*A* = 67° 22′; then as the sine of this angle: to PERP. *Ae* : : so radius (1.): to BA = 11. And as the sine of the angle CBA: to the PERP. 12 : : so radius: to BC = 13. Moreover; if the angle A, be multiplied by the side AC 20, it will give fC = 12; this divided by BC = 13, gives the supplement of the angle ABC = .923077 = 67° 23′; and *Ae* divided by AB gives the same. Consequently fC divided by the sine of the supplement of the angle ABC gives the side BC.

414. If the diameter of a certain wheel, having 8 arms, be 20ft. 4in.; what will be the length of a string that will reach from the out end of one arm to the out end of another?



As 113 : 355 : : 244 : 766$\frac{62}{113}$, circum. Then $\sqrt{2\,\overline{AC}^2}$ = 172.534+ =AB; the half of this is 86.267+ = AD or DB. Again, $\sqrt{\overline{AD}^2+\overline{AC}^2}$ = DC = 86.267+; which taken from 122, leaves DE = 35.733. Consequently, $\sqrt{\overline{DE}^2+\overline{AD}^2}$ = AE = 93.3742+ *Ans.**

414 *a.* A maple 104 feet high, standing on a horizontal plain, breaks and hangs on the stump, so that the top rests on the ground at 52 feet from the upright part; required its height.

Let *a* represent the height of the tree, and *b* the base or distance from it; then $\frac{a^2-b^2}{2a}$ = 39, the answer.

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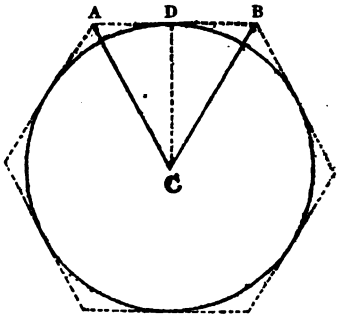
* Or, *by* NATURAL SINES, as CAD (67° 30′) = .923879 : 122 : : ACE (45°) = .7071067 : AE = 93.374+ as before. In like manner, if we have the chord, or the side of any regular inscribed polygon given, we may find the radius by the following proportion:

As the length of the chord, (or the side of the polygon,) : is to the SINE OF THE ANGLE at the centre : : so is the THE SINE OF THE ANGLE between the chord and either radius: to the LENGTH of the radius. ☞ In this case, the angle at the centre will always be given, because it will be some known part of 360 degrees: and the other two angles will be given, because, (since the sum of the angles of any triangle is equal to 180 degrees,) either of these will evidently be ONE HALF THE DIFFERENCE between the angle at the centre and 180°.

415. If the DIAMETER OF A DOLLAR be 1.54 inches, how many may be laid on an acre without doubling?



If the dollars be laid compactly, six dollars will touch one; so that each dollar will occupy a space equal to its circumscribing hexagon. Therefore,

As the cosine BCD (60°) = .8660254 : to DC = .77 : : so is the sine of BCD (30°) = .5 : to DB = .44456; which multiplied by DC (= .77) gives the area of the whole triangle ABC = .342311. This multiplied by 6, the number of such triangles, gives .2053866, the area of the hexagon. Hence 6272640, the square inches in an acre, divided by the area of the hexagon, gives 3054065, the number of dollars required.

MISCELLANEOUS QUESTIONS.

1. FROM NINETY TAKE FORTY,
FROM FORTY TAKE TEN;
SUBTRACT SIX FROM SIXTY,
AND WHAT REMAINS THEN? *Ans.* 134.

2. What is the difference between six dozen dozen, and half a dozen dozen? *Ans.* 792.

3. Jacob by contract was to serve Laban for his two daughters 14 years; and after he had accomplished 11 years, 11 months, 11 weeks, 11 days, 11 hours, and 11 minutes, how long had he to serve?

Ans. 1*y.* 11*mo.* 3*w.* 2*d.* 12*h.* 49*m.*

4. Just fifteen pair of ladies' gloves
For ninety dimes had I;
How many pair of that same kind
Will forty eagles buy? *Ans.* 666.+

5. What part of 9 pence is $\frac{2}{3}$ of 7 pence? *Ans.* $\frac{14}{18}$.

6. When the air presses with its full weight, in very fair weather, it may be demonstrated, that there presses upon a human body about 33905 pounds of that fluid matter; and in foul weather, when the air is most light, only 30624 pounds: what difference of weight?

Ans. 3281 *pounds.*

7. If the third of eight be four,
What is the fifth of twenty more? *Ans.* $8\frac{4}{5}$.

8. What part of 3d. is 3 cents? *Ans.* $\frac{24}{25}$.

9. How far from the end of a stick of timber 30 feet long, must a lever be placed, so that two men may each carry as much at the lever, as a third man at the end of the stick?

Divide the length of the stick by the number of men; and half the quotient will be the length required. *Ans.* 5*ft.*

10. A woman bought a certain number of eggs at 2 for a cent, and as many at 3 for a cent; and sold them all out at the rate of 5 for 2 cents, and by so doing lost 4 cents; how many eggs had she? *Ans.* 240.

11. A harmless dove was soaring high,
To stretch her wings in space—
At length a hawk did her espy,
And gave the dove a chase;
Just forty chains were then between
Those birds as we could view—

While the poor dove flew seventeen,
The hawk just twenty-two;
The hawk pursued with all his strength,
As those who saw did say—
Then tell the chains he flew in length,
Before he caught his prey. *Ans.* 17ft.

12. A man sold cloth at 11s. 6d., whereby he gained 15 per cent.; but finding it better than any in market at that price, he raises it to 12s.; I demand what he gained per cent. by his last sale? *Ans.* 20 *per cent.*

13. The largest of the Egyptian pyramids is square at the base, and measures 693 feet on a side; how much ground does it cover?
Ans. 11 *acres* 40*rds.*

14. The wheel of a perambulator turns just once and a half in a rod; what is its diameter? *Ans.* 3½ *feet.*

15. A country clown addressed a charming belle,
Who in both wit and learning did excel;
The youth, unskilled in numbers, as will show,
Desirous was the lady's age to know—
When she replied with a majestic air,
With piercing words peculiar to the fair,
"My age is such if multiplied by three
Two-sevenths of that product tripled be—
The square root of two-ninths of that is four,
Now tell my age or never see me more." *Ans.* 28 *years.*

16. Sold goods for $588, and by so doing lost 16 per cent., whereas in dealing I ought to have cleared 25 per cent. How much did I sell them under their real value? *Ans.* $287.

17. Agreed for a platform to the curb of a round well, at 7½d. per foot: the inward part, round the mouth of the well, is 36 inches in diameter, and the breadth of the platform to be 15½ inches; what will it come to?
Ans. 10*s.* 10*d.* 2.4*qrs*

18. A horse in the midst of a meadow suppose,
Made fast to a stake by a line from his nose,
How long must this line be, that feeding all round,
Permits him to graze just an acre of ground? *Ans.* 117 *ft.* 9*in.*

19. Laid out in a lot of muslin $500, but upon examination 2 parts in 7 proved to be damaged, so that I could make but 5s. 6d. per yard of it. and by so doing lost $53; how must I sell the undamaged muslin per ell English to clear 50 per cent. on the whole? ($1 = 8s.) *Ans.* 15s.

20. Required the difference between the area of a circle, whose radius is 50 yards, and its greatest inscribed square. *Ans.* 2654 *square yards.*

21. There is a section of a tree 25 inches over; I demand the difference of the areas of the inscribed and circumscribed squares, and how much they differ from the area of the section.

Ans. 312.5, difference of the squares; the former is 178.375 inches LESS and the latter 134.125 MORE than the section.

22. I am constrained to plant a grove,
To entertain the nymph I love;
This simple grove I must compose,
Of nineteen trees in nine straight rows,
And in each row five trees dispose:
The distance of each outside tree,
That's in this grove five rods must be;
Then tell the length of golden wire,
The CIRCLE OF IT will require? *Ans.* 55.845 *rods.*†

23. A gay young fellow soon got the better of ⅔ of his fortune; he then gave £1500 for a commission, and his profusion continued till he had but £450 left, which he found to be just ⅜ of his money, after he had purchased his commission; what was his fortune at first? *Ans.* £3780.

24. Edmund I can place four 3's, so as to make precisely 7 too?

25. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot; what would be the weight of such a tube, that would extend across the Atlantic, estimating the distance at 3000 miles?

Ans. 1*lb.* 8*oz.* 6*pwt.* $3\frac{9}{13}$*grs.*

26. Three men bought a grindstone of 50 inches diameter; I demand how much of its diameter each must grind off to have equal shares, if one first grind his share and then the next, until the stone is ground away?*

Ans. { 1st must grind 9.175172 inches of the diameter.
2d .... 11.957315 ....
3d .... 28.867513 ....

27. A round mash tub is 45 inches at the top and 36 at the bottom, and its perp. height is 50 inches; how many beer gallons will it hold?

Ans. 229.3531.

28. One hundred dollars you divide
Between two worthy men,
That when each part is fairly squared,
The difference is but ten. *Ans.* $50.05, and $49.95.

29. A younger brother received £1560, which was just $\frac{7}{12}$ of his elder brother's fortune, and $5\frac{3}{8}$ times the elder brother's fortune was $\frac{3}{8}$ as much again as the father was worth; pray, what was the value of his estate?

Ans. £19165 14*s.* $3\frac{3}{4}$*d.*

30. A certain tower on one side of a street is 249, and another on the opposite side is 219 feet; now, allowing the street to be 396 feet broad, whereabout in it must a ladder be placed, so as to reach the top of both towers? *Ans.* $180\frac{3}{11}$ *from the highest tower.*

31. A and B can do a piece of work in 5 days; A can do it in 7 days; in what time can B do it? *Ans.* $17\frac{1}{2}$ *days.*

32. Suppose two wheels, the one 12 feet, the other 12ft. 4in. diameter, be placed on an axletree 8 feet long, and set rolling on a plane till they describe a circle; how much ground will the larger wheel circumscribe?

Ans. 102.744*sq. rods.*

33. Old John who had in credit lived,
Tho' now reduced, a sum received;
This lucky hit's no sooner found,
Than clam'rous duns came swarming round;
To th' landlord—baker—many more,
John paid in all, pounds ninety-four.
Half what remained—a friend he lent,
On Joan and self one-fifth had spent;
And when of all these sums bereft,
One-tenth of th' sum received had left;
Now show your skills, ye learned youths,
And by your work the sum produce. *Ans.* £141.

34. A & B are on opposite sides of a circular field 268 rods about; they begin to go round it, both the same way, at the same time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes; how many times will they go round the field, before the swifter overtakes the slower?

Ans. A will go $16\frac{1}{2}$. *and B* 17 *times.*

35. A fellow said when he counted his nuts 2 by 2, 3 by 3, 4 by 4, 5 by 5, and 6 by 6, there was still an odd one; but when he told them 7 by 7, they came out even: how many had he? *Ans.* 721.

36. A, B, & C can complete a piece of work in 12 days; A can do it alone in 23 days, and B in 37; in what time can C do it? *Ans.* $77\frac{125}{181}$.

37. Required the side of a cubical box that shall contain a ton of air?

Ans. 30.6+ *feet.*

* Square the diameter, and divide the square by the number of men—subtract the quotient from the square, and the square root of the remainder will be the diameter of the stone, after the first has ground his share.

38. A landed man two daughters had,
And both were very fair;
To each he gave a piece of land,
One round the other square:
At forty shillings to an acre,
Each piece its value had;
The money which did compass each,
For it exactly paid;
If 'cross a shilling be an inch,
(As it is very near;)
Which had the better fortune; she
Who had the round, or square?
The number also is required
Of acres each did own;
And now, good sir, you are desired
To make those numbers known.*

Ans. The one who had the square had the better fortune by £26922 5*s.* 5*d.* The square contains 63726.4, and the round 49265.31456 acres.

39. A & B cleared, by an adventure at sea, 45 guineas, which was £35 *per cent.* upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to 4, as often as 8 to B; what sum did each adventure?

Ans. *A* £104 4*s.* $2\frac{2}{19}$*d.*, *B* £75 15*s.* $9\frac{17}{19}$*d.*

40. I would put 20hhd. of London beer into 10 wine pipes, and desire to know what the cask must hold, that will receive the difference.

Ans. $47\frac{11}{17}$ *beer gallons.*

41. Suppose a light-house built on the top of a rock; the distance between the place of observation and that part of the rock level with the eye 620 yards; the distance from the top of the rock to the place of observation 846 yards, and from the top of the light-house 900 yards; the height of the light-house is required. *Ans.* 76.77*yds.*

42. A wretch, who spurned the virtuous path below,
To heaven above by other means would go:
He by mechanic art a ladder raised,
Its height the vulgar and the learn'd amazed;
Thus Jacob did, said he, and why not I
Scale heaven as well as he? at least I'll try.—
He mounts his ladder, rapidly ascends,
And bids adieu to all terrestrial friends.
Now wrapped in clouds unpierced by human eyes,
And now environed with the azure skies.
But now a ball red-hot he lets fall down,
Which fires a cannon at the base, whose sound
Ascends his ear, in thrice the length of time
The ball was falling from this height sublime.
Ye sages tell if he to heaven has got,
Or how far soaring from this earthly spot.

Ans. 733592 *feet* = 139 *miles*+

43. What is the difference between half a solid foot, and half a foot solid? *Ans.* 648 *inches.*

44. Suppose a perfectly straight trough, 2000 miles long, (its sides being extended at pleasure and ends open,) to be placed on this globe; and suppose water be admitted at the centre; to what depth will the water obtain before it will begin to run out at each end? (earth's diam. 7964m.)

Ans. 123.6 *miles*+.

45. A lent B a solid stack of hay, measuring 20 feet every way; sometime after B returned one measuring 10 feet every way: what part of the hay remains due? *Ans.* $\frac{1}{8}$.

* The side of the square and the diameter of the circle are equal.

46. Suppose the diameter of the base of a conical ingot of gold to be 3 inches, and its height 9; what length of wire may be expected from it, without loss of metal, the diameter of the wire being $\frac{1}{100}$ part of an inch?
Ans. 4*m.* 83*rds.* $\frac{7}{11}$.

47. On the FOURTH OF JULY a pole was erected,
Composed of six pieces, and nicely connected;
Two feet and six inches it measured around,
On the place where it stood at the top of the ground;
The form was a cone in surface complete,
The height of the same was twice sixty feet:
What length of inch riband, procured at the shop,
Will wind round the pole from bottom to top,
And have it lay smooth, and plain to be seen,
By leaving a space of five inches between? *Ans.* 936 *feet.*

48. When 11 swaths, each 6 feet wide, were mown about a square meadow, one-half the grass was cut; how many acres were in said meadow?
Ans. 4.6627415294329.

49. A had a beam of oak timber, 18 inches square throughout, and 25 feet long; which he bartered with B, for an equilateral triangular beam of the same length, each side 24 inches; required the balance, at 16*d.* per foot? *Ans.* 17*s* $3\frac{3}{10}$*d.*

50. A person, being asked the time of day, answered, that it was between 4 and 5, and that the hour and minute hands were exactly together; required the time. *Ans.* $21\frac{9}{11}$*m. after* 4.

51. A cistern has 2 pipes to supply it: by the first, it may be filled in 45 minutes, and by the second in 55; it has likewise a discharging pipe, by which it may, when full, be emptied in 30 minutes: now, if these 3 pipes be all left open, in what time will the cistern be filled?
Ans. 2*h.* 21*m.* $25\frac{5}{7}$*s.*

52. I drop'd a ball from Jackson's bridge,
Its height above the stream to tell—
A pendulum two feet in length,
Made six vibrations while it fell;
Can you in numbers fairly show
How high this lofty bridge must be
Above the stream that glides below,
The feet and inches show to me? *Ans.* 215*ft.* 10.54*in.*

53. A, B, and C are to share £100, in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively; but C dying, it is required to divide the whole sum properly between the other two.
Ans. A must have £57 2*s.* $10\frac{2}{7}$*d.*, *and B* £42 17*s.* $1\frac{1}{7}$*d.*

54. Bought a horse, which I sold again for $56, and gained as much per cent. as the horse cost me; what did I give for it? *Ans.* $40.

55. If 12 oxen will eat $3\frac{1}{3}$ acres of grass in four weeks, and 21 oxen will eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being allowed to grow uniformly?

$$\text{As} \left\{ \begin{matrix} 12 : 21 \\ 4 : 9 \end{matrix} \right\} :: 3\tfrac{1}{3} : 13\tfrac{1}{3}. \quad \text{Then } 13\tfrac{1}{3} - 10 = 3\tfrac{1}{3} \textit{ acres},$$

increase of 3 acres in 5 weeks. Whence,

$$\text{As} \left\{ \begin{matrix} 10 : 24 \\ 5^{*} : 14^{\dagger} \end{matrix} \right\} :: 3\tfrac{1}{3} : 21 \text{ acres.} \quad \text{Then } 21 + 24 = 45 \text{ acres.} \quad \text{Again,}$$

$$\text{As} \left\{ \begin{matrix} 18 : 4 \\ 3\tfrac{1}{3} : 45 \end{matrix} \right\} :: 12 : 36 \text{ oxen, } \textit{Ans.}$$

56. A person would hew a stick of timber that is 12 inches square, into an octagonal shape; where must he strike the lines?
Ans. $3\frac{1}{2}$ *inches from each corner.*

* Equal to 9 weeks — 4 weeks. † Equal to 18 weeks — 4 weeks.

57. I placed a bowl into the storm,
To catch the drops of rain—
A half a globe was just its form,
Two feet across the same;
The storm was o'er, the tempest past,
I to the bowl repaired—
Six inches deep the water stood,
It being measured fair;
Suppose a cylinder, whose base
Two feet across within,
Had stood exactly in that place,
What would the depth have been? *Ans.* 2.5 *inches.*

58. A general, disposing his army into a square battalion, found he had 231 over and above, but increasing each side with one soldier, he wanted 14 to fill up the square; how many had he? *Ans.* 19000.

59. There are three horses, belonging to different men, employed as a team to draw a load of salt from Newburyport to Boston, for £4 10*s.*: A and B are supposed to do $\frac{3}{11}$ of the work; A and C $\frac{5}{13}$; and B and C $\frac{4}{13}$ of it; they are to be paid proportionally: can you divide it as it should be?

Ans. A *must have* £1 15*s.* $5\frac{560}{611}$*d.*
B .... £0 16*s.* $7\frac{64}{611}$*d.*
C .... £1 17*s.* $11\frac{320}{611}$*d.*

60. Suppose a horse fastened to a post three feet in circumference, by a rope 113 yards long; how many miles will he have to travel to wind up the rope? *Ans.* 22*m.* $6\frac{15}{44}$*fur.*

61. If, during ebb tide, a wherry should set out from Haverhill to come down the river, and, at the same time, another should set out from Newburyport, to go up the river, allowing the distance to be 18 miles; suppose the current forwards the one, and retards the other, $1\frac{1}{2}$ miles per hour: the boats are equally laden, the rowers equally good, and, in the common way of working in still water, would proceed at the rate of 4 miles per hour: where in the river will the two boats meet?

Ans. 12 *m.* 3*fur. from Haverhill.*

62. There is an island 50 miles in circumference, and three men start together to travel the same way about it; A goes 7 miles per day, B 8, and C 9: when will they all come together again? *Ans. in* 50 *days.*

63. Suppose a man 6 feet high to travel round the globe; how much larger will the circle be described by his head, than that described by his feet? *Ans.* 37.699 *feet.*

64. Just rising from the sea I've seen,
When stationed in the shrouds.
The lofty peak of Teneriffe,
That penetrates the clouds—
Just fifty leagues from it was I,
My reckoning being true,
And from the water to my eye,
The feet was eighty-two;
Suppose these observations just,
To make the question brief,
Above the level of the sea,
How high is Teneriffe? *Ans.* 2.4234 *miles.*

65. A, B, and C bought a house for $1000; says A to B, C, give me $\frac{1}{2}$ of your money and I can pay for the house; says B to A, C, give me $\frac{1}{3}$ of your money and I can do so too; says C to A, B, give me $\frac{1}{4}$ of your money and I alone can pay for the house: how many dollars had each?

Ans. A 626\frac{18}{107}$, B 719\frac{67}{107}$, and C 775\frac{74}{107}$.

66. I have four weights in my possession, which together make forty pounds with which I can weigh from one pound to forty: required the weights? *Ans.* 1, 3, 9, and 27.

67. Divide one shilling among three men, and give the second $\frac{1}{7}$ more than the first, and the last $\frac{1}{7}$ more than the second.

Ans. $2\frac{34}{37}d$, $3\frac{33}{37}d$, and $5\frac{7}{37}d$.

68. A father dying, left a sum in cash, to be divided among his four sons, in such a manner, that A's share together with $\frac{1}{3}$ of the other three; B's with $\frac{1}{4}$ of the other three; C's with $\frac{1}{5}$ of the other three; and D's with $\frac{1}{6}$ of the other three, may always make \$137. What was each son's share?

Ans. A's \$47, B's \$77, C's \$92, and D's \$101.

69. A merchant has four kinds of cloth, marked A, B, C, D;

E bot. of A 24 *yds.*, of B 6 *yds.*, of C 12 *yds.*, & of D 3 *yds.*, = \$123;
F bot. of A 6 . . ., of B 9 . . ., of C 3 . . ., & of D 12 . . ., = \$132;
G bot. of A 18 . . ., of B 12 . . ., of C 4 . . ., & of D 6 . . ., = \$116;
H bot. of A 30 . . ., of B 6 . . ., of C 9 . . ., & of D 4 . . ., = \$121;

What is the price of a yard of each kind?

Ans. A \$1, B \$3, C \$5, and D \$7 *per yard.*

70. One evening I chanced with a tinker to sit,
Whose tongue ran a great deal too fast for his wit;
He talked of his art with abundance of mettle;—
So I asked him to make me a flat bottom'd kettle:
Let the top and the bottom diameters be,
In just such proportion as five is to three;—
Twelve inches the depth I proposed, and no more:
And to hold in ale gallon seven less than a score:
He promised to do it and straight to work went;
But when he had done it he found it too scant—
He altered it then, but too big he had made it;
For though it held right, its diameters failed it;
Thus making it often too big and too, little,
The tinker at last had quite spoiled his kettle:
But he swears he will bring his said promise to pass,
Or else that he'll spoil every ounce of his brass.
Now to keep him from ruin, I pray find him out
The diameters' length, for he'll ne'er do it I doubt.

Ans. The bottom diameter is 14.640171, *and top diameter* 24.400285 *inches.*

71. A, B & C / B, C & D / C, D & A / D, A & B } can intrench a field in { 12 / 14 / 16 / 18 } days; { How long will it take each man to perform the work alone? and how long if they all work together?

Ans. A can do it in $51\frac{15}{60}$ days, B in $35\frac{7}{45}$ days, C in $28\frac{28}{107}$ days, D in $131\frac{1}{11}$ days, and ALL together in $10\frac{274}{2775}$ days.

72. A man dying, left his wife with child, and by his will ordered that if she went with a son, $\frac{5}{8}$ of the estate should belong to him, and $\frac{1}{3}$ to her: and if she had a daughter, he appointed the mother $\frac{2}{3}$ and the daughter $\frac{1}{3}$: but it happened she was delivered of both, a son and a daughter, by which she lost in equity £2000 more than if it had been only a daughter: what would have been her dowry, if she had had only a son?

Ans. £1750.

73. The famous TUN of Heidleburgh, that being heretofore annually replenished with rhenish, had in it some wine that was many ages old, before the French demolished it in the late war; it was 31 feet in length, and 21 in diameter, and pretty nearly cylindrical: pray how much did it contain? *Ans.* 318 *tuns*, & 183.8 *gallons.*

74. Our Satellite the moon, is a globe in diameter 2170 miles; I require to know how many quarters* of wheat she would contain, if hollow; and how much yard wide stuff would make her a waistcoat, were she to be clothed.

Ans. { 791061152810212 8540.06 *quarters of wheat.*
{ 45824284391424 *square yards of stuff.*

* Eight bushels.

75. Suppose one farthing had been put out, at 6 per cent. per annum compound interest, at the commencement of the Christian era; what would it have amounted to in 1784 years:* and suppose the amount to be in standard gold, allowing a cubic inch to be worth £53 2*s.* 8*d.*, how large would the mass have been?

Ans. { *It would have amounted to* £1496716346568748209435714551509890767065361 11*s.* 3½*d.*; *and the mass would have been more than* 4159308999840288 *times larger than the globe we inhabit.*

76. If a solid globe of glass at the furnace, whose diameter is 8 inches, be blown into a hollow globe until the shell be but $\frac{1}{10}$ of an inch in thickness; what will then be its diameter, and how much wine will it hold?

Let a represent the diameter of the globe, and b the thickness of the shell; then $\frac{\sqrt{a^3 - 2b^3}}{6b} - b$, will give the diameter of the cavity $= 29.1118$ nearly; and it will hold 49*galls.* 2*qts.* 0.8*pts.*

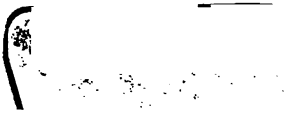
77. Suppose the natural increase of mankind to be such that they double once in 50 years; and suppose a general resurrection had taken place *Anno Mundi* 5800; what would have been the number of souls?—and suppose the whole globe of the world, sea and land, rocks and mountains, to be equally divided among them, how large a part would each have drawn for a body?

$\frac{2^{116} - 1}{2 - 1} \times 2 = 166\,15,359873,114284,112975,882535,043070$ *souls.*

Then 672733373088547413688320000, the cubic inches in the globe, divided by the number of souls above, gives .00000000405 + of a cubic inch which each soul would have drawn for a body; which is less than the thirteenth part of the smallest particle that can be seen by the naked eye.

N. B. Divide a cubic inch into eight million small cubes, and one of those cubes may be seen by a good eye without the help of a magnifier.

78. Let there be two lights at the distance of 90 yards from each other: and the less light to be to the greater as 1 to 4; required the point, between those lights, where a body would be equally illuminated by them, according to this known principle in physics, that the effect of a light is four times greater, when it is twice nearer; or increases as the square of the distance decreases.

Let $90 = a$, $1 = b$, $4 = c$; and let x be the distance of the less light from a point taken at pleasure in the line which joins the two lights: then $a - x$ will be the distance of the greater light from the same point. The square of the two distances will be x^2 and $(a-x)^2$; whence it follows, that if the lights were of equal force, the effects they would produce in this same point, would be to each other as $\frac{1}{x^2}$ to $\frac{1}{a-x^2}$. But the absolute quantities of those lights being to each other in the ratio of b to c, their effects will be to each other as $\frac{b}{x^2}$ to $\frac{c}{a-x^2}$. Now that the point taken at pleasure, may become the point required, we put these two points equal to each other. Thus, $\frac{b}{x^2} = \frac{c}{a-x^2}$; whence, m being put $= c - b\ (= 3)$, we have $x = \frac{\sqrt{ba^2m + b^2a^2} \pm ba}{m} = 90$, and 30; that is, there are two points where a body would be equally illuminated by the said lights; one 30 yards distant from the less light, in the line which joins the lights; and the other 90 yards from the same light, in the same line produced on the opposite side.

* The principal doubles once in 11.89 years, nearly. (See page 139.)